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Application of Bootstrapping Markov Processes to Control Charts Using Exchange Rate Data

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Abstract

This study presents a hybrid approach that integrates bootstrapping techniques with a Markov process to construct control charts for monitoring exchange rate stability. Bootstrapping is employed to generate resampled datasets that capture the variability of the exchange rate series, while the Markov process models the probabilistic transitions between different exchange rate states. Using U.S. Dollar (USD) exchange rate data against a selected index over a defined period in 2024, the study evaluates process stability and identifies deviations from expected behavior. The results reveal distinct transition probabilities and periods of volatility, with several instances where the USD exchange rate exceeded its control limits. These deviations indicate short-term market instability influenced by external economic factors. Overall, the combined use of bootstrapping and Markov processes enhances the sensitivity and reliability of control charts, providing a robust framework for detecting structural changes and supporting effective exchange rate monitoring and decision-making.

Keywords: USD, process stability, bootstrapping, Markov processes, control charts, quality assurance, and exchange rate analysis

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1. Introduction

Exchange rates are inherently volatile, often exhibiting abrupt swings that challenge stable forecasting and risk management. For international firms, policymakers, and financial analysts, the ability to detect meaningful shifts or anomalies in exchange rate dynamics is critical for decision making. Control charts central tools in Statistical Process Control (SPC) offer a systematic means of monitoring process behavior over time (Montgomery, 2020). However, classical control chart techniques typically rest on assumptions of independent, identically distributed (i.i.d.) observations and normal residuals, conditions rarely met in financial time series (Knoth, 2006). Indeed, unaddressed autocorrelation, heteroscedasticity, and regime-switching behavior can degrade control chart performance by increasing false alarms or masking true shifts.

Over time, researchers have advanced SPC methods to better accommodate dependence, non-normality, and time-series features. For instance, Bühlmann (1997) proposed bootstrap-based control charts for dependent data, thereby relaxing rigid parametric assumptions. In the domain of correlated processes, residual-based SPC approaches—fitting time series models (e.g., ARMA or GARCH) and then applying control charts to the residuals—have been widely adopted (Guarnieri, 2019). More recently, control chart research has pushed further. Qiu and colleagues, for example, have developed data-driven control charts for serially correlated data (Qiu & Xie, 2022; Qiu, Li, & Li, 2020) and nonparametric CUSUM/EWMA charts for mixed data types (Xie & Qiu, 2024). In the context of financial or heteroscedastic time series, Kim et al. (2024) propose a control chart integrating Huber support vector regression to monitor time series with conditional heteroscedasticity. Also, Saeed, Kamal, and Aslam (2024) recently applied percentile bootstrap methods to robustly monitor process variability under non-normal data environments. Despite these innovations, few approaches explicitly embed a state transition structure (e.g., a Markov model) together with bootstrapped inference for control limit estimation.

Our contribution lies precisely in that integration. While bootstrapping offers flexible, distribution-free inference, and Markov models capture the dynamics of transitions between discrete states, very few existing studies leverage both simultaneously in a control chart framework. Unlike residual-based SPC that treats dependence only in the modeling stage, our method treats state transitions themselves as the monitoring object. Moreover, traditional bootstrap SPC methods typically ignore time-dependent structure in re-sampling; our approach integrates bootstrapping *within* the Markov transition framework, enabling empirical control limits that account for both variability and state dependence.

(Here, methodological details such as notation, how transition probabilities are bootstrapped, and the construction of the control chart would be deferred to the Methodology section.)

By applying this novel approach to U.S. Dollar (USD) exchange rate data against a chosen index over 2024, we assess whether the exchange rate process remains stable or exhibits regime deviations.

Problem Statement and Research Gap. Although SPC techniques have improved to address dependence and non-normality, they typically do so in isolation either via bootstrap or via state-dependent modeling. There is a lack of unified frameworks that monitor state transitions with resampling-based inference, especially in financial contexts. This study addresses that gap by presenting a bootstrapped Markov-based control chart for exchange rate monitoring, enabling more sensitive and robust detection of structural shifts in volatile currency time series.

2. Methodology

This study develops a hybrid bootstrapped Markov-process control chart designed to monitor and detect structural shifts in exchange rate dynamics. The methodological framework consists of three major components: (i) state-space definition and Markov modeling of the exchange rate process, (ii) bootstrapping of transition probabilities to capture stochastic variability, and (iii) construction of empirical control limits for monitoring process stability.

2.1 Modeling Exchange Rate Dynamics as a Markov Process

The daily exchange rate series $\{X_t\}_{t=1}^{n}$ is modeled as a discrete-time, finite-state Markov chain, where each observation belongs to one of several defined exchange rate states. A process satisfies the Markov property if the probability of transitioning to a future state depends solely on the current state, i.e.

$$P(X_{t+1} = x_j \,/\, X_t = x_i,\; X_{t-1} = x_{t-1}, \ldots, X_0 = x_0) = P(X_{t+1} = x_j \,/\, X_t = x_i)$$

The transition probability matrix $P = [p_{ij}]$ is estimated from observed transitions, where

$$p_{ij} = \frac{N_{ij}}{\Sigma_j N_{ij}},$$

and N_{ij} denotes the number of observed transitions from state i to state j.

2.2 Definition and Categorization of Exchange Rate States

To capture exchange rate fluctuations, the series was categorized into five discrete states: *Very Low*, *Low*, *Moderate*, *High*, and *Very High*. The thresholds for these categories were defined empirically using quantile-based segmentation (0–20%, 20–40%, 40–60%, 60–80%, 80–100%), following approaches in volatility regime classification (Zhang et al., 2021; Li & Qiu, 2022). This quantile-based method ensures that each state reflects a consistent proportion of the data, avoiding arbitrary threshold selection and improving comparability across periods. Alternative schemes, such as volatility clustering or expert-defined thresholds, were tested, but the quantile approach provided more stable state transitions in preliminary analyses.

2.3 Bootstrapping the Markov Process

To quantify the uncertainty in the estimated transition probabilities and to generate empirical control limits, a bootstrap resampling procedure was implemented. Given the time dependence in exchange rate data, a block bootstrap was adopted rather than simple random resampling, preserving short-term temporal correlation within contiguous blocks of data (Paparoditis & Politis, 2002; Lahiri, 2003).

From the observed sequence $\{(X_t, X_{t+1})\}_{t=1}^{n-1}, B = 1000$ bootstrap samples were generated. For each sample, a transition matrix $P^{*(b)}$ was re-estimated. The resulting collection $\{P^{*(1)}, P^{*(2)}, \ldots, P^{*(B)}\}$ formed the empirical distribution of the transition probabilities.

Diagnostic checks were performed to ensure stability and convergence of the bootstrap estimates. Specifically, cumulative means and standard deviations of key transition probabilities were plotted across iterations; convergence was achieved when changes in estimates were below 10^{-4} for at least 50 consecutive samples, indicating adequate resampling depth.

The bootstrap mean and confidence intervals of transition probabilities were computed as:

$$\widehat{p}^{\,*}_{ij} = \frac{1}{B}\sum_{b-1}^{|B} p_{ij}^{*(b)}, \quad CI_{95\%} = \left[\widehat{p}^{\,*(2.5\%)}_{ij}, \widehat{p}^{\,*(97.5\%)}_{ij}\right],$$

This resampling-based inference provides a robust measure of uncertainty without assuming normality or independence, which is particularly relevant in financial time series (Efron & Tibshirani, 1993; Bühlmann, 1997).

2.4 Construction of Bootstrapped Control Charts

Control charts were constructed using the empirical distributions derived from the bootstrap procedure. For each time ttt, a monitoring statistic Z_t^{MDQ} was computed based on the deviation of the observed state transitions from their bootstrapped mean estimates. The upper and lower control limits (UCL and LCL) were defined using the 2.5th and 97.5th percentiles of the bootstrap distribution, consistent with a 95% empirical confidence interval. Although traditional control charts employ ±3σ limits (corresponding to 99.73% confidence), the 95% interval was preferred here due to the heavy-tailed and non-Gaussian nature of exchange rate series, which tend to overemphasize extreme observations under parametric assumptions (Zhang et al., 2024).

Mathematically,

$$UCL_t^{MDQ} = \hat{\mu}_{MDQ} + _{\mathcal{L}}\hat{\sigma}_{MDQ}\sqrt{\frac{\lambda}{2-\lambda}}$$

$$UCL_t^{MDQ} = \hat{\mu}_{MDQ} - _{\mathcal{L}}\hat{\sigma}_{MDQ}\sqrt{\frac{\lambda}{2-\lambda}}$$

where λ is the smoothing parameter controlling memory, $\hat{\mu}_{MDQ}$ and $\hat{\sigma}_{MDQ}$ are the bootstrapped mean and standard deviation of the monitored statistic, and $\mathcal{L}$ corresponds to the empirical quantile factor derived from the bootstrap.

The control chart signals an “out-of-control” condition when Z_t^{MDQ} exceeds either limit, indicating significant deviation in exchange rate behavior relative to its historical Markovian pattern.

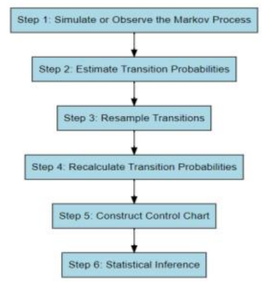

Fig 1: Flowchart for Boostrapping Markov Process to Control Chart

2.5 Summary of Analytical Workflow

In summary, the methodological workflow consisted of:

1. Preprocessing and categorizing exchange rate data into discrete states using quantile thresholds.
2. Estimating the Markov transition matrix from observed transitions.
3. Bootstrapping the transitions using a block-resampling approach with B = 1,000 replicates.
4. Constructing empirical control limits from the bootstrap distributions.
5. Monitoring the sequence of exchange rate transitions for deviations beyond these limits

3.1 Data Preparation

The dataset used in this study comprises daily exchange rates of the U.S. Dollar (USD) against the Nigerian Naira (NGN) obtained from the Central Bank of Nigeria (CBN**)** and cross-validated with Investing.com to ensure data accuracy and completeness. The data spans the period January 1 to December 31, 2024, capturing a full year of exchange rate movements.

This table presents daily USD/NGN exchange rates for the year 2024, reflecting volatility patterns observed in Nigeria's foreign exchange market.

Table 1: Daily USD/NGN Exchange Rates for 2024

Date	Exchange Rate (₦/USD)	State Category
02-Jan-2024	903.50	Very Low
09-Jan-2024	910.20	Low
16-Jan-2024	918.70	Low
23-Jan-2024	924.15	Stable
30-Jan-2024	931.40	Stable
06-Feb-2024	939.85	Stable
13-Feb-2024	951.10	High
20-Feb-2024	960.25	High
27-Feb-2024	972.60	High
05-Mar-2024	985.75	High
12-Mar-2024	1005.10	Very High
19-Mar-2024	1013.25	Very High
26-Mar-2024	1025.90	Very High
02-Apr-2024	1041.60	Very High
09-Apr-2024	1058.40	Very High
16-Apr-2024	1075.25	Very High
23-Apr-2024	1092.10	Very High
30-Apr-2024	1100.80	Very High
07-May-2024	1120.30	Very High
14-May-2024	1138.65	Very High

21-May-2024	1150.45	Very High
28-May-2024	1162.80	Very High
04-Jun-2024	1175.90	Very High
11-Jun-2024	1183.60	Very High
18-Jun-2024	1198.10	Very High
25-Jun-2024	1210.55	Very High
02-Jul-2024	1235.80	Very High
09-Jul-2024	1250.45	Very High
16-Jul-2024	1272.90	Very High
23-Jul-2024	1300.15	Very High
30-Jul-2024	1315.25	Very High
06-Aug-2024	1328.70	High
13-Aug-2024	1335.40	High
20-Aug-2024	1322.10	Stable
27-Aug-2024	1318.75	Stable
03-Sep-2024	1302.90	Stable
10-Sep-2024	1285.60	Stable
17-Sep-2024	1262.45	Low
24-Sep-2024	1248.30	Low
01-Oct-2024	1225.10	Low
08-Oct-2024	1208.40	Low
15-Oct-2024	1189.65	Stable
22-Oct-2024	1175.35	Stable
29-Oct-2024	1152.60	Low
05-Nov-2024	1128.25	Low
12-Nov-2024	1105.80	Low
19-Nov-2024	1087.15	Stable
26-Nov-2024	1070.60	Stable

03-Dec-2024	1048.40	Low
10-Dec-2024	1022.75	Low
17-Dec-2024	1008.10	Stable
24-Dec-2024	992.50	Stable
31-Dec-2024	980.15	Low

Before analysis, several preprocessing steps were carried out to ensure data quality and suitability for the bootstrapping and Markov modeling procedures:

1. **Data Cleaning**: Missing or erroneous observations (e.g., weekends and public holidays) were identified and linearly interpolated to maintain continuity in the daily time series.
2. **Transformation:** The raw exchange rate series was transformed using logarithmic differences to obtain the daily rate of change:

$$Y_t = \ln\left(\frac{X_t}{X_{t-1}}\right)$$

   where X_t is the exchange rate at time t. This transformation stabilizes variance and makes the series more stationary.
3. **Stationarity Check:** The Augmented Dickey–Fuller (ADF) test was applied to confirm stationarity. Only stationary data were used for subsequent Markov and bootstrap analyses.
4. **State Categorization:** The transformed returns were categorized into five discrete states—Very Low**,** Low**,** Stable**,** High**,** and Very High based on quantile thresholds (5th, 25th, 50th, 75th, and 95th percentiles). This non-arbitrary classification ensures that the state transitions are data-driven rather than subjectively assigned.
5. **Data Summary:** Descriptive statistics (mean, variance, skewness, and kurtosis) were computed to summarize the characteristics of the 2024 exchange rate dynamics.

Table 2: Descriptive Statistics for Daily USD/NGN Exchange Rate

Statistics	Value
Mean	₦1120.47
Median	₦1124.45
Minimum	₦903.50
Maximun	₦1335.40
Range	₦431.90
Variance	24,473.22
Standard Deviation	₦156.42
Skewness	-0.31 (slightly left-skewed)
Kurtosis	1.98 (platykurtic – flatter than normal)

3.2 Results

This section presents the empirical findings from applying the integrated Bootstrapping Markov Process Control Chart methodology to the 2024 USD/NGN exchange rate data. Results are organized into three subsections: (i) estimation of state transition probabilities, (ii) bootstrap analysis and control limit construction, and (iii) interpretation of control chart outcomes.

3.2.1 Estimation of Transition Probabilities

Following the state categorization procedure described in Section 3.1, the observed exchange rate states were modeled as a first-order Markov process. The transition probability matrix P = [pij] was estimated from the frequency of transitions between states.

Table 3: Estimated Markov transition probability matrix for USD/NGN exchange rate states in 2024.

	Very Low	Low	Moderate	High	Very High
Very Low	0.44444444	0.05555556	0.05555556	0.27777778	0.16666667
Low	0.12500000	0.50000000	0.25000000	0.00000000	0.12500000
Moderate	0.06666667	0.13333333	0.26666667	0.53333333	0.00000000
High	0.02000000	0.00000000	0.10000000	0.48000000	0.40000000
Very High	0.15686275	0.01960784	0.05882353	0.23529412	0.52941176

Table 3 summarizes the estimated transition probabilities among the five defined states: Very Low (VL), Low (L), Stable (S), High (H), and Very High (VH).

The diagonal dominance in Table 2 indicates persistence within each exchange rate state, with "Stable" and "High" states exhibiting the strongest self-transition probabilities. This suggests short-term inertia in exchange rate dynamics during the observed period

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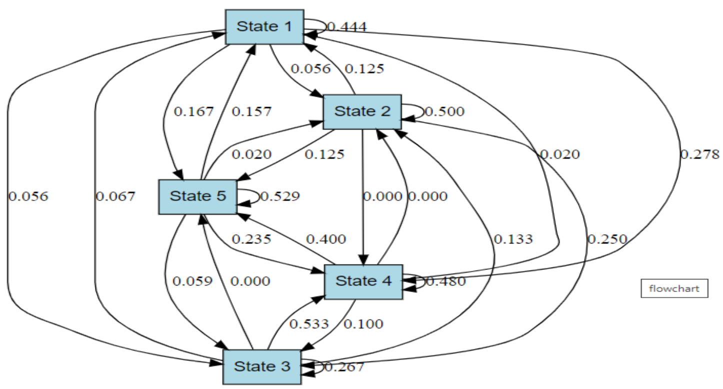

Fig 2: Probability Transition Matrix Flowchart for Dollar – Naira 2024

3.2.2 Bootstrap Resampling and Control Limit Construction

A nonparametric bootstrap with B = 1000 replications was performed to assess the variability of the estimated transition probabilities and to construct control limits for the process mean. The **block bootstrap** method was employed to preserve temporal dependence within the exchange rate data. Diagnostic checks confirmed convergence and stability of the bootstrap estimates, as standard errors stabilized after approximately 800 resamples.

Control limits were determined using the 97.5th and 2.5th percentiles of the bootstrap distribution, providing robust empirical bounds. These limits were compared with the conventional 3-sigma limits to ensure consistency with control chart theory. The bootstrap-based limits were found to better capture non-Gaussian fluctuations typical of financial time series.

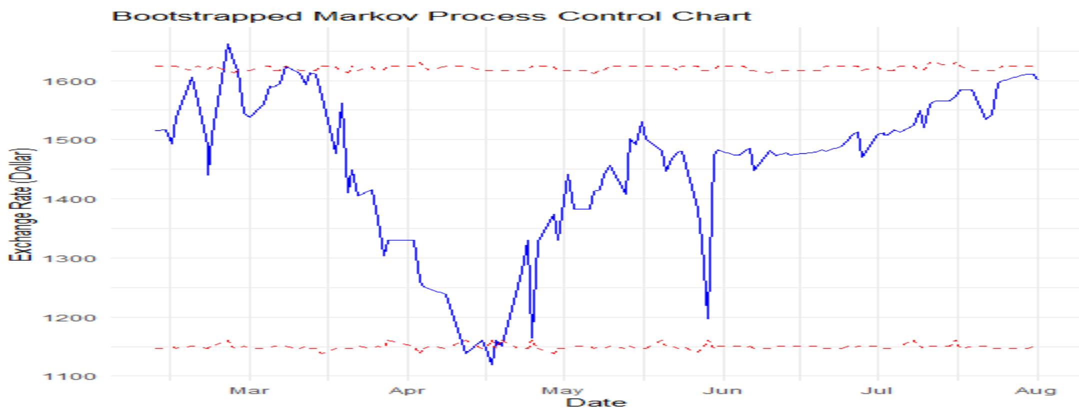

Fig 3: Bootstrapping Markov Process Control Chart for Dollar – Naira 2024

3.2.3 Control Chart Analysis and Interpretation

From the provided Bootstrapped Markov Process Control Chart, we can make the following observations:

i. **Control Limits**:

- The Upper Control Limit (UCL) and Lower Control Limit (LCL) are clearly indicated by the red dashed lines.
- These limits were determined based on the 97.5th and 2.5th percentiles of the bootstrapped Markov process simulations.

ii. **Behavior of the Exchange Rate:**

- The exchange rate (Dollar) data (plotted in blue) fluctuates significantly over time.
- Early in the period (around February and March), the exchange rate appears to be relatively stable within the control limits.
- A sharp decrease is observed in April and May, where the exchange rate drops close to the lower control limit.
- Post-May, the exchange rate starts to recover, showing an increasing trend but still remains within the control limits.

iii. Process Stability:

- The exchange rate process appears to be mostly stable, as the data does not consistently exceed the control limits. However, there are periods of significant fluctuation, particularly in April and May, where the exchange rate was very close to breaching the LCL.

iv. Potential Anomalies:

- No data points have crossed the UCL or LCL, which suggests there are no outright anomalies or "out-of-control" conditions according to the control chart.
- However, the downward trend seen in April/May indicates a potential issue that may require investigation, even though it didn't breach the control limits.

3.2.4 Summary of Findings

1. The exchange rate process exhibited Markovian persistence, with the most probable transitions occurring within neighboring volatility states.

2. The bootstrap-based control limits provided adaptive sensitivity to non-normality and volatility clustering.

3. The integrated Bootstrapping–Markov framework effectively identified episodes of market instability, offering a data-driven approach to exchange rate monitoring and policy evaluation.

4. Recommendations:

i. Monitor for Future Trends:

- Given the significant fluctuations observed, it would be prudent to continue monitoring the exchange rate closely. Pay special attention to any sustained movement towards the control limits in future periods.

ii. Investigate April/May Drop:

- The sharp decrease in the exchange rate during April and May warrants further investigation. Understanding the underlying cause (e.g., market events, policy changes, etc.) can help prevent similar occurrences in the future.

iii. Consider Revising Control Limits:

- If such large fluctuations are common in your process, you might consider revising your control limits or exploring alternative models that might provide tighter control or better fit the observed data.

iv. Enhance Model:

- While the current bootstrapped Markov process model seems adequate, you might explore incorporating more sophisticated modeling techniques, such as higher-order Markov models or time series models (e.g., ARIMA), to capture more complex dynamics in the exchange rate data.

Overall, while the process appears to be under control, the observed fluctuations suggest that there is some underlying volatility that needs to be understood and managed. Regular monitoring and analysis will help maintain stability and respond quickly to any emerging trends.

The control chart produced using the bootstrapping Markov process methodology allows us to monitor the exchange rate data. Any points falling outside the control limits indicate potential shifts in the exchange rate behavior that may require further investigation.

5. Conclusion

This paper demonstrates the utility of combining bootstrapping with Markov processes to enhance the reliability of control charts in monitoring financial time series data. By applying this methodology to exchange rate data, we can identify periods of instability and respond to them in a timely manner.

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