Journal of Scientific Research and Development (2025) Vol. 24 (1) 84-110

JSRD Journal of Scientific Research & Development

A bi-annual journal published by the Faculty of Science, University of Lagos, Nigeria
http://jsrd.unilag.edu.ng/index.php/jsrd

Design of group acceptance sampling plans for truncated life tests using the percentiles of Zech distribution: Applications in industrial and medical settings

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(Received 21 May 2025/Revised 12 August 2025/Accepted 18 August 2025}

Abstract

Acceptance sampling is a quality technique used to minimize the risk of defective items reaching consumers. Unlike process-monitoring tools such as control charts, acceptance sampling focuses on the inspection of incoming materials and finished goods rather than continuous process evaluation. Traditionally applied in industrial settings, recent research has extended its application to survival analysis through the use of specialized statistical distributions. This study proposes a Group Acceptance Sampling Plan (GASP) based on the Zech distribution. The plan employs truncated life tests to improve both cost and time efficiency. The proposed method is evaluated using arbitrary parameter values, simulated and real-life datasets. Its performance is compared with group acceptance sampling plans based on the New Weibull-Pareto and Gompertz distributions, The results indicate that the Zech-based GASP requires the smallest number of groups, making it a more efficient and cost-effective option for both industrial quality control and survival analysis applications.

Key Words: Acceptance Sampling Plans, Consumer's Risk, Operating Characteristic Function, Probability distributions, Producer's Risk.

1.0 Introduction

Acceptance sampling plan (ASP) is one of the oldest statistical techniques in quality control. One of the most notable early frameworks, MIL-STD-105, was developed in 1942 as a United States defense standard, providing systematic procedures and tables for sampling by attributes (Harry et al., 1942). ASP is not performed during production but may be applied before production to decide on incoming batches or after production to evaluate the final product (Rao, 2009; Rao et al., 2013; Al-Omari and Al-Nasser, 2019; Alashaari, 2021; Ahmed and Yousof, 2021; Algarni, 2022; Gomes and Figueiredo, 2023).

In general, ASP involves selecting a sample from a lot, inspecting its quality characteristics, and deciding on the lot's acceptance or rejection based on the results. ASP can be based on attributes or variables. Attribute plans assess whether a product conforms or does not conform to quality standards, while variable plans measure continuous characteristics such as weight, hardness, or lifetime. When lifetime is the focus, the problem becomes a life test, which may be truncated to reduce time and cost while ensuring a specified mean or percentile life (Rao et al., 2012).

Probability distributions play a central role in the design of ASP, enabling calculation of acceptance probabilities and ensuring efficient decision-making (Alashaari, 2021). The choice of quality parameter depends on the distribution's symmetry: which make use of the means for symmetric distributions, while percentiles are more suitable for skewed ones.

A group acceptance sampling plan (GASP) is designed for simultaneous testing of multiple items, thereby reducing time and cost (Rao, 2009). Previous studies have applied GASP to various distributions, including the Exponentiated Fréchet (Rao et al., 2019), Weibull-Fréchet (Ahmed and Yousof, 2021), New Compounded Three-Parameter Weibull (Algarni, 2022), and others. However, acceptance sampling plans based on inverted compound distributions remain underexplored. This study develops a GASP for truncated life tests using the percentiles of the Zech distribution to address this gap.

2.0 Materials and Methods

This study develops a group acceptance sampling plan (GASP) for truncated life tests based on the 25th, 50th, and 75th percentiles of the Zech distribution. Simulated datasets are employed to demonstrate the performance of the proposed plans. Various combinations of parameter values were used to illustrate their general behaviour under different conditions. To evaluate practical applicability, two real-life datasets were analyzed; one from an industrial context and the other from a survival study. These datasets provide an empirical basis for assessing the effectiveness of the proposed Zech-based plans in both manufacturing and reliability scenarios. Furthermore, the performance of the developed plans is compared with other similar lifetime distributions.

2.1 Zech Distribution

The Zech distribution proposed by Adeyeye et al. (2023) is the reciprocal of Gompertz Inverse-Exponential distribution developed by Oguntunde et al. (2018). The cumulative distribution function and the probability density function (pdf) of Zech distribution are stated in equations (1) and (2), respectively.

$$G(t) = e^{\frac{\lambda}{\delta}\left\{1-\left[1-e^{-\theta t}\right]^{-\delta}\right\}} \; ; \; t > 0, \quad \theta > 0, \lambda > 0, \delta > 0 \qquad (1)$$

$$g(t) = \lambda\theta e^{-\theta t}\left[1-e^{-\theta t}\right]^{-\delta-1} e^{\frac{\lambda}{\delta}\left\{1-\left[1-e^{-\theta t}\right]^{-\delta}\right\}}; \quad t > 0, \theta > 0, \lambda > 0, \delta > 0 \qquad (2)$$

λ and δ are the shape parameters while θ is the scale parameter.

2.2 Quantile Function and Median of *Zech* distribution.

Quantile function is very important for generating random numbers which can be used for simulation studies. Aside that, it can also be used for finding the quantiles i.e. quartiles, octiles, deciles and percentiles of a distribution, which are necessary for deriving the measures of skewness and kurtosis.

The quantile function of Zech distribution is given in equation (3)

$$Q(q) = -\frac{1}{\theta}\left\{\ln\left[1-\left(1-\frac{\delta}{\lambda}\ln q\right)^{-\frac{1}{\delta}}\right]\right\} \qquad (3)$$

Where $q \sim Uniform\ (0,1)$.

2.3 Failure Probability of Zech distribution.

The failure probability, p , plays a crucial role in acceptance sampling plans as it helps estimate the likelihood of accepting a batch or lot that contains defects. Understanding failure probabilities allows quality control managers to make informed decisions about whether to accept or reject a batch, based on the acceptable level of risk. The probability, p, represents the chance that an item fails before time t_0 (Aslam et al., 2010).

Before the experiment time t_0, the probability of failure of products under the Zech distribution is given by

$$p = e^{\frac{\lambda}{\delta}\left\{1-\left[1-e^{-\theta t_0}\right]^{-\delta}\right\}} \quad (4)$$

Recall that the quantile function of Zech distribution is

$$t_q = -\frac{1}{\theta}\left\{\ln\left[1-\left(1-\frac{\delta}{\lambda}ln\, q\right)^{-\frac{1}{\delta}}\right]\right\} \quad (5)$$

Setting

$$\xi_q = -\left\{\ln\left[1-\left(1-\frac{\delta}{\lambda}ln\, q\right)^{-\frac{1}{\delta}}\right]\right\} \quad (6)$$

Therefore,

$$t_q = \frac{\xi_q}{\theta} \quad (7)$$

The experiment is stopped at the time t_0 indicated by

$$t_0 = at_q^0 \quad (8)$$

where t_0 is the termination time, a is the termination ratio while t_q and t_q^0 are the true and specified quantile life of the product respectively. The scale parameter θ can be expressed as

$$\theta = \frac{\xi_q}{t_q} \quad (9)$$

By substituting $\frac{\xi_q}{t_q}$ for θ in equation (4), the probability of failure of Zech distributed items is obtained as follows

$$p = exp\left(\frac{\lambda}{\delta}\left\{1-\left[1-exp\left(\frac{-a\xi_q}{\left(t_q/t_q^0\right)}\right)\right]^{-\delta}\right\}\right) \quad (10)$$

Then in expanded form, p becomes,

$$p = exp\left(\frac{\lambda}{\delta}\left\{1-\left[1-exp\left(aln\left[1-\left(1-\frac{\delta}{\lambda}lnq\right)^{-\frac{1}{\delta}}\right]\left(\frac{t_q}{t_q^0}\right)^{-1}\right)\right]^{-\delta}\right\}\right) \quad (11)$$

The failure probability, p, is independent of the scale parameter, θ. When time is scaled appropriately, the influence of the scale parameter, is effectively neutralized, leaving p

dependent only on the shape of the distribution and the relative time to failure.

2.4. Operating Procedures for Group Acceptance Sampling Plans

The procedures as stated by Aldossary et al. (2021), are as follows:

i. The number of groups g are selected and predefined r items are allocated to each group so that the sample size for a lot will be $n = gr$

ii. Select the acceptance number c for a group and specify the experiment time, t_0.

iii. Perform the experiment for the g groups at the same time and record the number of failures for each group.

iv. Accept the lot if the number of failures in each of all the groups does not exceed the acceptance number c by the experiment time.

v. The experiment is terminated as soon as more than c failures occur in any group and the lot is rejected.

The lot's acceptance probability for the group ASP is given as

$$L(p) = \left[\sum_{d=0}^{c} \binom{r}{d} p^r (1-p)^{r-d}\right]^g \quad (12)$$

The consumer's risk is often expressed by the consumer's confidence level. If the confidence level is P^*, then the consumer's risk is $\beta = 1 - P^*$ (Aslam & June, 2009). The number of groups in the proposed sampling plan will be determined so that the consumer's risk does not exceed β.

The minimum number of groups required must satisfy the inequality $L(p) \leq 1 - P^*$

To determine the design parameters g and c for a given quality level, along with specific consumer's and producer's risks, the following optimization problem needs to be solved.

Minimize the number of groups,

Subject to the constraints

$P_a(P_1) \leq \beta$

$P_a(P_2) \geq 1 - \alpha$

$g \geq 1, c \geq 0$

$$P_a\left(p_1 | \frac{t_q}{t_q^0} = r_1\right) = \left[\sum_{d=0}^{c} \binom{r}{d} {p_1}^d (1-p_1)^{r-d}\right]^g \leq \beta \quad (13)$$

$$P_a\left(p_2 | \frac{t_q}{t_q^0} = r_2\right) = \left[\sum_{d=0}^{c} \binom{r}{d} {p_2}^d (1-p_2)^{r-d}\right]^g \geq 1 - \alpha \quad (14)$$

The quantile ratios of the consumer's and producer's risks are represented by r_1 and r_2, respectively. The probabilities of failure to be utilized are provided in equations (9) and (10)

$$p_1 = exp\left(\frac{\lambda}{\delta}\left\{1 - \left[1 - exp\left(aln\left[1 - \left(1 - \frac{\delta}{\lambda} lnq\right)^{-\frac{1}{\delta}}\right]\right)\right]^{-\delta}\right\}\right) \quad (15)$$

$$p_2 = exp\left(\frac{\lambda}{\delta}\left\{1 - \left[1 - exp\left(aln\left[1 - \left(1 - \frac{\delta}{\lambda} lnq\right)^{-\frac{1}{\delta}}\right]\left(\frac{t_q}{t_q^0}\right)^{-1}\right)\right]^{-\delta}\right\}\right) \quad (16)$$

The quantile ratio, $\frac{t_q}{t_q^0}$ whose values are 2, 4, 6, 8 and 10 is considered at the risk of the producer. This will make it more likely that people will accept the high quality product while the quantile ratio $\frac{t_q}{t_q^0} = 1$ is considered at the consumer' risk to ensure that the product of poor quality is rejected.

2.5. Designing the Group ASP for the Percentiles of Zech Distribution Using Simulated Data

A simulation study was carried out for comparing the results arrived in the above illustration. The simulated results were based on 10,000 runs using R programming. A random sample of size, n = 50 is simulated from Zech distributions with the shape parameters γ *and* δ are specified as 0.5 and 0.5 respectively, and the scale parameter, θ,is given as 1.5. The resulted simulated data, arranged in an ascending order are given below:

0.02778655, 0.03317616, 0.04735757, 0.06004381, 0.06114307, 0.06284239, 0.06826930, 0.06904844, 0.07298172, 0.09098504, 0.10769026, 0.11148058, 0.13567743, 0.14090141, 0.14185104, 0.15214113, 0.16586403, 0.17557878, 0.17835086, 0.19288063, 0.20177095, 0.20434227, 0.22674940, 0.22976317, 0.23249269, 0.26384114, 0.27018619, 0.29298173, 0.30543376, 0.30790196, 0.31092006, 0.34486852, 0.35415989, 0.37646817, 0.41019261, 0.56431109, 0.58716907, 0.58789835, 0.58904138, 0.68091964, 0.68175724, 0.78287974, 0.81122984, 0.83194921, 1.04402310, 1.27194805, 1.45457358, 1.53540965, 2.34016071, 3.04635964

The following estimates of parameters of Zech distribution, fitted on the simulated data were found: $\hat{\lambda} = 0.2876$ and $\hat{\delta} = 0.7260$

The estimated parameters were used to develop the Group ASP for the truncated life tests based on 25th percentiles of Zech distribution. The following values were assumed for consumer's risks $\beta = 0.25, 0.10, 0.05, 0.01$ while the producer's risk was kept at $\alpha = 0.05$. The values to be considered for termination ratios were ***a***= 0.5, 0.7 and 1.0, while $\boldsymbol{r}$ (the number of items in each group) is given as either 5 or 10. The result of the study is presented in Table 1

2.6. Designing the Group ASP for the Truncated Life Tests Based on the Percentiles of Zech Distribution Using Arbitrary Parameter Values

To provide valuable insights into the performance and effectiveness of the proposed acceptance sampling plans under different scenarios, thereby helping organizations make informed decisions about quality control strategies, the following combinations of shape parameters of Zech distribution are used: $\gamma = 0.5, \delta = 0.5$. The Group acceptance sampling plans are developed for the 50th percentile Zech distribution.

2.7. Comparative Study on the Proposed Acceptance Sampling Plans.

The effectiveness of the proposed acceptance sampling plans is compared with the group acceptance sampling plans , we developed for the truncated life tests based on New Weibull-Pareto distribution proposed by Suleman and Albert (2015), and group acceptance sampling plan, based on Gompertz distribution introduced by Harsh Tripathi et al., (2022) to develop Skip-lot Sampling Plan with Reinspection (SkSP-R). The comparison is done using both industrial and survival datasets at different percentiles. The cumulative distribution function (cdf) and q^{th} quantile of New Weibull-Pareto distribution are given in the equations below.

$$G(t) = 1 - e^{-\delta\left(\frac{t}{\theta}\right)^{\lambda}}, t > 0, \delta > 0, \theta > 0, \lambda > 0 \qquad (17)$$

$$t_q = \theta\left\{\frac{1}{\delta}\ln\left(\frac{1}{1-q}\right)\right\}^{\frac{1}{\lambda}} \qquad (18)$$

2.8. The Failure Probability of New Weibull-Pareto distribution

The probability of failure of New Weibull-Pareto distribution is derived as follows

$$p = 1 - e^{-\delta\left(\frac{t_0}{\theta}\right)^{\lambda}}, t > 0, \delta > 0, \theta > 0, \lambda > 0 \qquad (19)$$

Let

$$\xi_q = \left\{\frac{1}{\delta}\ln\left(\frac{1}{1-q}\right)\right\}^{\frac{1}{\lambda}} \qquad (20)$$

$$t_q = \theta\xi_q \qquad (21)$$

$$\theta = \frac{t_q}{\xi_q} \qquad (22)$$

$since\ t_0 = at_q^0,$

$$p = 1 - e^{-\delta\left(a\xi_q\left(\frac{t_q}{t_q^0}\right)^{-1}\right)^{\lambda}} \qquad (23)$$

The failure probability, p, of the New Weibull-Pareto distribution, in expanded form is given as

$$p = 1 - exp\left[-\delta\left(a\left\{\frac{1}{\delta}ln\left(\frac{1}{1-q}\right)\right\}^{\frac{1}{\lambda}}\left(\frac{t_q}{t_q^o}\right)^{-1}\right)^{\lambda}\right] \qquad (24)$$

2.9. Failure Probability of Gompertz distribution

The cumulative distribution function (cdf) and q^{th} quantile of Gompertz distribution are given below

$$F(t) = 1 - e^{-\lambda\left(e^{\frac{t}{\theta}}-1\right)} \qquad (25)$$

$$t_q = \theta\ln\left[1 - \frac{1}{\lambda}ln(1-q)\right] \qquad (26)$$

The failure probability, p, of Gompertz distribution is derived as follows

$$p = 1 - e^{-\lambda\left(e^{\frac{t_0}{\theta}}-1\right)}, \qquad t > 0, \theta > 0, \lambda > 0 \qquad (27)$$

Let

$$\xi_q = \ln\left[1 - \frac{1}{\lambda} ln(1-q)\right] \quad (28)$$

$$t_q = \theta\xi_q \quad (29)$$

$$\theta = \frac{t_q}{\xi_q} \quad (30)$$

$since\ t_0 = at_q^0,$

$$p = 1 - e^{-\lambda\left(e^{a\varepsilon_q\left(\frac{t_q}{t_q^0}\right)^{-1}} - 1\right)} \quad (31)$$

The failure probability of Gompertz distribution, in expanded form is

$$p = 1 - exp\left[-\lambda\left(exp\left(a\ln\left[1 - \frac{1}{\lambda} ln(1-q)\right]\left(\frac{t_q}{t_q^0}\right)^{-1}\right) - 1\right)\right] \quad (32)$$

3.0 Results

The results of the group Acceptance Sampling Plan (ASP) based on simulated data were presented in Table 1. Table 2 provides the results of the group ASP applied to arbitrary parameter values. Tables 3 and 4 detail the descriptive statistics of the industrial data, along with the parameter estimates, selection criteria, and goodness-of-fit measures for the data. The proposed group ASP results using the small electric cart data are displayed in Table 5.

Tables 6 and 7 present the descriptive statistics and parameter estimations of the Zech distribution using COVID-19 data, while Table 8 showcases the proposed group ASP applied to survival data. To evaluate the effectiveness of the proposed group ASP compared to the group ASPs based on the Gompertz and New Weibull-Pareto distributions, Table 9 summarizes the shape parameter estimates for all three distributions across the two datasets. Comparative study results for the industrial and survival datasets are shown in Tables 10 and 11, respectively.

Figures 1 and 2 demonstrate that the Zech distribution fits both the industrial and survival data well. Figure 3 highlights that the operating characteristic (OC) curve of the proposed ASP exhibits the steepest slope, providing more accurate results compared to the Gompertz and New Weibull-Pareto distributions.

Table 1: Group ASP for Zech Distribution when $\gamma = 0.2876$ and $\hat{\delta} = 0.7260$ Showing the Minimum g and c at 25th Percentile Using Simulated Data

		$r = 5$									$r = 10$								
		$a = 0.5$			$a = 0.7$			$a = 1.0$			$a = 0.5$			$a = 0.7$			$a = 1$		
β	r_2	g	c	P_a	g	c	P_a	g	c	P_a	g	c	P_a	g	c	P_a	g	c	P_a
	2	23	1	0.9602	50	2	0.9769	89	3	0.9799	31	2	0.9916	7	2	0.9664	6	3	
	4	4	0	0.9871	2	0	0.9662	4	1	0.9930	2	0	0.9871	1	0	0.9662	1	0.9600	
0.25	6	4	0	0.9991	2	0	0.9956	1	0	0.9863	2	0	0.9991	1	0	0.9956	1	1	
	8	4	0	0.9999	2	0	0.9993	1	0	0.9967	2	0	0.9999	1	0	0.9993	1	0.9924	
	10	4	0	1.0000	2	0	0.9999	1	0	0.9992	2	0	1.0000	1	0	0.9999	1	0	
																		0.9727	
																		0	
																		0.9935	
																		0	
																		0.9983	
	2	448	2	0.9893	82	2	0.9625	147	3	0.9671	51	2	0.9862	43	3	0.9864		4	
	4	6	0	0.9806	13	1	0.9985	6	1	0.9895	3	0	0.9806	4	1	29		0.9794	
0.10	6	6	0	0.9986	3	0	0.9934	2	0	0.9727	3	0	0.9986	2	0	0.9979	2	1	
	8	6	0	0.9999	3	0	0.9990	2	0	0.9935	3	0	0.9999	2	0	0.9912	1	0.9849	
	10	6	0	1.0000	3	0	0.9998	2	0	0.9983	3	0	1.0000	2	0	0.9987	1	0	
																0.9998	1	0.9727	
																		0	
																		0.9935	
																		0	
																		0.9983	
	2	583	2	0.9861	106	2	0.9518	191	3	0.9575	66	2	0.9821	56	3	0.9824		4	
	4	7	0	0.9775	17	1	0.9980	7	1	0.9877	4	0	0.9743	5	1	37		0.9738	
0.05	6	7	0	0.9984	4	0	0.9912	3	0	0.9594	4	0	0.9981	2	0	0.9974	3	1	
	8	7	0	0.9998	4	0	0.9987	3	0	0.9903	4	0	0.9998	2	0	0.9912	3	0.9774	
	10	7	0	1.0000	4	0	0.9998	3	0	0.9975	4	0	1.0000	2	0	0.9987	2	1	
																0.9998	2	0.9990	
																		0	
																		0.9871	

																		0
																		0.9966
	2	896	2	0.9788	0	5	0.0000	0	5	0.0000		2	0.9728	85	3	0.9733	57	4
	4	11	0	0.9648	25	1	0.9971	11	1	101		0	0.9617	8	1	0.9958	4	0.9599
0.01	6	11	0	0.9974	6	0	0.9869	11	1	0.9807	6	0	0.9972	3	0	0.9869	4	1
	8	11	0	0.9998	6	0	0.9980	4	0	0.9992	6	0	0.9997	3	0	0.9980	2	0.9699
	10	11	0	1.0000	6	0	0.9996	4	0	0.9871	6	0	1.0000	3	0	0.9996	2	1
										0.9966	6							0.9986
																		0
																		0.9871
																		0
																		0.9966

Table 1 shows the table pattern of the truncated life test on group acceptance sampling plan based on the 25th percentiles of Zech distribution using simulated data.

From Table 1, the following are observed.

1. As the consumer's risk, β, decreases, the number of groups increases. When the consumer's risk is 0.01, the plan requires more number of groups to be selected for testing. This is a clear testament to the effectiveness of the plan. The stringiest condition that must be met by a good sampling plan is, to maintain a very small consumer's risk, a larger number of groups must be tested.

(ii) As the quantile ratio increases, the probability of acceptance of the lot approximates to almost 1.

TABLE 2. Group ASP for Zech Distribution Using Arbitrary Parameter values, $\gamma = 1.0$ and $\delta = 0.2$ to Show the Minimum g and c at 50th Percentile

		$r = 5$									$r = 10$								
		$a = 0.5$			$a = 0.7$			$a = 1.0$			$a = 0.5$			$a = 0.7$			$a = 1$		
β	r_2	g	c	P_a	g	c	P_a	g	c	P_a	g	c	P_a	g	c	P_a	g	c	P_a
0.25	2	0	5	0.0000	199	4	0.9641	0	5	0.0000	14	4	0.9528	11	5	0.9615	8	5	0.9560
	4	11	2	0.9913	5	2	0.9831	3	2	0.9596	2	2	0.9839	1	2	0.9689	1	2	0.9774
	6	3	1	0.9853	2	1	0.9715	1	1	0.9602	1	1	0.9794	1	2	0.9940	1	2	0.9741
	8	3	1	0.9945	2	1	0.9884	1	1	0.9823	1	1	0.9922	1	1	0.9760	1	1	0.9919
	10	1	0	0.9556	2	1	0.9946	1	1	0.9911	1	1	0.9965	1	1	0.9884	1	1	0.9638
0.10	2	0	5	0.0000	0	5	0.0000	0	5	0.0000	85	5	0.9690	64	6	0.9732	41	7	0.9733
	4	18	2	0.9858	8	2	0.9730	12	3	0.9895	4	2	0.9681	3	3	0.9878	2	3	0.9553
	6	5	1	0.9756	3	1	0.9575	4	2	0.9889	2	1	0.9593	2	2	0.9881	1	2	0.9741
	8	5	1	0.9909	3	1	0.9827	2	1	0.9650	2	1	0.9844	1	1	0.9760	1	2	0.9919
	10	5	1	0.9960	3	1	0.9919	2	1	0.9822	2	1	0.9930	1	1	0.9884	1	1	0.9638
0.05	2	0	5	0.0000	0	5	0.0000	0	5	0.0000	111	5	0.9597	83	6	0.9654	54	7	0.9650
	4	24	2	0.9811	10	2	0.9664	15	3	0.9869	5	2	0.9603	4	3	0.9838	2	3	0.9553
	6	6	1	0.9707	10	2	0.9942	5	2	0.9861	2	1	0.9593	2	2	0.9881	2	3	0.9937
	8	6	1	0.9891	4	1	0.9770	2	1	0.9650	2	1	0.9844	2	1	0.9525	2	2	0.9839
	10	6	1	0.9952	4	1	0.9892	2	1	0.9822	2	1	0.9930	2	1	0.9770	1	1	0.9638
0.01	2	0	5	0.0000	0	5	0.0000	0	5	0.0000	894	6	0.9757	641	7	0.9787	427	7	0.9785
	4	36	2	0.9718	15	2	0.9500	23	3	0.9800	16	3	0.9899	6	3	0.9758	5	3	0.9829
	6	9	1	0.9565	15	2	0.9914	7	2	0.9807	7	2	0.9913	4	2	0.9763	3	2	0.9906
	8	9	1	0.9837	5	1	0.9714	7	2	0.9945	3	1	0.9766	2	1	0.9925	2	1	0.9839
	10	9	1	0.9928	5	1	0.9865	3	1	0.9735	3	1	0.9895	2	1	0.9770	2	1	0.9941

Table 2 presents the group acceptance sampling plans for the truncated life test based on the 25th percentile lifetime of Zech distribution, using the arbitrary parameter values.

3.1 From Tables 2, the following are observed:

1. As the consumer's risks decrease, the number of groups increase.
2. There is an inverse relationship between termination ratio, a, and the number of groups, g, especially when 'a' increases from 0.5 to 0.7
3. (ii) As the quantile ratio increases, the probability of acceptance of the lot approximates to almost 1.

3.2 Application of the Proposed Plan to Industrial Data

The dataset represents the lifetime in months, of 20 small electric carts used by a manufacturing company for internal transportation and delivery services in a large manufacturing facility. The data has been used previously by Al-Nasser and Ahsan-ul-Haq, (2021) for developing acceptance sampling plans from a truncated life test based on Power Lomax distribution. The observations are as follows: 0.9, 1.5, 2.3, 3.2, 3.9, 5.0, 6.2, 7.5, 8.3, 10.4, 11.1, 12.6, 15.0, 16.3, 19.3, 22.6, 24.8, 31.5, 38.1, 53.0

In this section, a group acceptance sampling plan for the truncated life test based on the percentiles of Zech distribution is designed, The descriptive statistics and the estimates of the parameters of Zech distribution, fitted on the lifetime data, as well as the goodness-of-fit criteria and P-value are presented below.

Table 3: Descriptive Statistics for data 1.

Min	1st Quartile	Median	Mean	3rd Quartile	Max.	Standard Deviation	Skewness	Kurtosis
0.900	4.725	10.750	14.675	20.125	53.000	13.6637	1.348706	4.279926

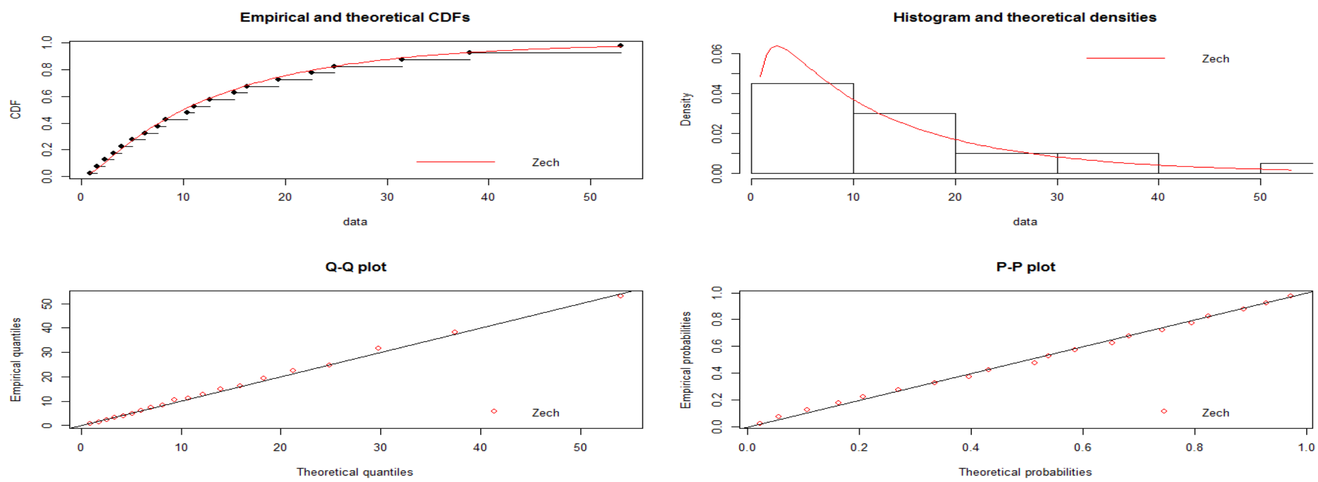
Table 3 shows that the data is skewed to the right. Interestingly, the shape of the pdf graph of *Zech* distribution is also positively skewed. This implies that *Zech* distribution is suitable for modelling the industrial data. Also, the Kurtosis value of 4.279926 suggests that the data is leptokurtic, having a kurtosis of 1.279926 above that of normal distribution which is 3.0

TABLE 4: MLE Estimates, SE, AIC, BIC, KS, AD, CVM with the P-Value for the lifetime data

γ	δ	θ	AIC	BIC	AD	P-Value
0.8474 (0.5699)	0.2856 (0.3964)	0.0652 (0.0280)	152.5674	155.5546	0.0767	0.9772

The results in Table 4 indicate that Zech distribution fits the lifetime data, having a P-value greater than 0.05 The standard errors are in the parenthesis.

The empirical and theoretical cdfs, Histogram and theoretical densities, Quantile-Quantile plot, and Probability-Probability plot are shown in figure 1. They all suggest that Zech distribution fits the data well.



From figure 1, it can be deduced that Zech distribution fits the empirical data well, judging from the P-P, Q-Q plot, as well as the graph of histogram and theoretical densities. Judging from the closeness of the lines to the points, it can be concluded that Zech distribution fits the dataset.

3.3 Determination of the True 75th Percentile Lifetime of the Electric Carts Using Zech Distribution

From the estimates of the parameters of Zech distribution, $\hat{\theta} = 0.0652, \hat{\lambda} = 0.8474, \hat{\delta} = 0.2856, q = 0.75$

The true qth quantile is given as $t_q = -\frac{1}{\theta}\left\{\ln\left[1-\left(1-\frac{\delta}{\lambda} ln\, q\right)^{-\frac{1}{\delta}}\right]\right\}$

$t_{0.75} = -\frac{1}{0.0652}\left\{\ln\left[1-\left(1-\frac{0.2856}{0.8474} ln\, 0.75\right)^{-\frac{1}{0.2856}}\right]\right\} = t_{0.75} = 19.7009$ months ~20 months.

The 1st 15 of the 20 small electric carts data have lifetimes of at most 20 months.

To test the accuracy of our result, Likewise, 75th percentile is where 0.75(20)th item lies =15th item. From the dataset on the small electric carts, the 15th item is 19.3 months.

From Table 3, the 75th percentile = 20.125. So we can conclude that the True 75th percentile lifetime of the industrial data is approximately 20 months.

Table 5: Group ASP for Zech when $\lambda = 0.8474$ and $\delta = 0.2856$ Showing Minimum g And c at 75th Percentile Using the Industrial Data

		$r = 5$									$r = 10$								
		$a = 0.5$			$a = 0.7$			$a = 1.0$			$a = 0.5$			$a = 0.7$			$a = 1$		
β	r_2	g	c	P_a	g	c	P_a	g	c	P_a	g	c	P_a	g	c	P_a	g	c	P_a
	2	-	-	-	-	-	-	-	-	-	8	6	0.9597	7	7	0.9525	5	8	
	4	3	2	0.9610	3	3	0.9870	2	3	0.9610	1	3	0.9783	1	4	0.9793	1	0.9523	
0.25	6	1	1	0.9631	2	2	0.9799	1	2	0.9635	1	2	0.9768	1	3	0.9843	1	5	
	8	1	1	0.9847	1	1	0.9576	1	2	0.9868	1	2	0.9935	1	2	0.9718	1	0.9732	
	10	1	1	0.9928	1	1	0.9781	1	2	0.9946	1	1	0.9705	1	2	0.9890	1	4	
																		0.9838	
																		3	
																		0.9783	
																		2	
																		0.9529	
	2	-	-	-	-	-	-	-	-	-		7	0.9708	38	8	0.9688		9	
	4	12	3	0.9900	5	3	0.9784	9	4	45		3	0.9572	2	4	40		0.9660	
0.10	6	4	2	0.9902	2	2	0.9799	3	3	0.9881	2	1	0.9768	1	3	0.9591	1	5	
	8	2	1	0.9697	1	1	0.9576	2	2	0.9896	1	2	0.9935	1	2	0.9843	1	0.9733	
	10	2	1	0.9857	1	1	0.9781	2	2	0.9738	1	2	0.9705	1	2	0.9718	1	4	
										0.9892	1					0.9890	1	0.9838	
																		3	
																		0.9783	
																		2	
																		0.9529	
	2	-	-	-	-	-	-	-	-	-		7	0.9625	49	8	0.9599		9	
	4	16	3	0.9867	7	3	0.9699	12	4	58		3	0.9572	2	4	52		0.9560	
0.05	6	5	2	0.9877	3	2	0.9700	3	3	0.9841	2	2	0.9542	1	3	0.9591	3	6	
	8	2	1	0.9697	3	2	0.9908	2	2	0.9896	2	2	0.9870	1	2	0.9843	1	0.9847	
	10	2	1	0.9857	2	1	0.9567	2	2	0.9738	2	1	0.9705	1	2	0.9718	1	4	
										0.9892	1					0.9890	1	0.9838	
																		3	
																		0.9783	
																		2	

																		0.9529	
	2	-	-	-	-	-	-	-	-	-		8	0.9764	543	9	0.9753	-	-	-
	4	24	3	0.9802	10	3	0.9572	17	4	472		4	0.9838	5	5	0.9825	4	6	
0.01	6	7	2	0.9828	4	2	0.9602	5	3	0.9776	5	2	0.9542	2	3	0.9689	2	0.9796	
	8	3	1	0.9548	4	2	0.9878	3	2	0.9827	2	2	0.9870	1	2	0.9718	1	4	
	10	3	1	0.9786	2	1	0.9567	3	2	0.9610	2	2	0.9957	1	2	0.9890	1	0.9680	
										0.9838	2							3	
																		0.9783	
																		2	
																		0.9529	

Table 5 presents the group ASP on the truncated life test based on the 75th percentile lifetime of Zech distribution using the data on the small electric carts.

For illustration, suppose the small electric cart manufacturer would prefer to employ the developed group acceptance sampling plan to assess whether the true 75th percentile lifetime of the electric carts whose lifetime follows Zech distribution with the shape parameters, $\hat{\lambda} = 0.8474$ and $\hat{\delta} = 0.2856$, surpasses a specified 75th percentile lifetime of the product. From the results in Table 5, the true 75th percentile lifetime of the electric carts is approximately 20 months, i.e. $t_q = 20$ months. Let the specified 75th percentile lifetime of the electric carts be 5 months, i.e. $t_q^0 = 5$ months, the quantile ratio or quality level, $r_2 = \frac{t_q}{t_q^0} = \frac{20}{5} = 4$. This could be interpreted as, the true 75th percentile lifetime of the electric carts is 4 times its specified 75th percentile lifetime.

If $r = 5$, $\alpha = 0.05$, $\beta = 0.25$, $a = 0.5$, and $r_2 = 4$; the optimal parameters chosen from Table 5 are: $g = 3, c = 2$, and the probability of acceptance of the lot = 0.9610. It means that a sample of 15 electric carts shall be divided into three groups consisting of 5 electric carts in a group will be selected from the batch. The sample size is calculated from $n = g \times r$.

The total sample size, $n = 5 \times 3 = 15$. In simpler terms, a sample size of 15 shall be drawn from the lot and divided into 3 groups, each consisting of 5 electric carts and subjected to a life test. If not more than two failures exist in any of the groups at the completion of the experiment time, it shall be statistically confirmed that the 75th percentile lifetime of the carts is greater than its specified 75th percentile lifetime, and the lots will be accepted. Otherwise, it will be rejected.

For the purpose of this real world illustration, from data 1, 5 small electric carts have a lifetime below 5 months. Hence, regarding the lifetime of the small electric carts in the population, the experimenter could advise the company that the 75th percentile lifetime of the small electric carts in a particular group is at a desirable level.

3.4 Application of the Proposed Plan to Survival Data

In this section, the application of group acceptance sampling plan for the truncated life tests based on the percentiles of Zech distribution to a survival data, representing the United Kingdom's COVID-19 deaths covering 76 days, from 15 April to 30 June 2020. The data has been used previously by Alsultan (2024) to design a group acceptance sampling plan based on truncated life tests using extended odd Weibull exponential distribution. The observations are as follows:

0.0587, 0.0863, 0.1165, 0.1247, 0.1277, 0.1303, 0.1652, 0.2079, 0.2395, 0.2751, 0.2845,0.2992, 0.3188, 0.3317, 0.3446, 0.3553, 0.3622, 0.3926, 0.3926, 0.4110, 0.4633, 0.4690,0.4954, 0.5139, 0.5696, 0.5837, 0.6197, 0.6365, 0.7096, 0.7193, 0.7444, 0.8590, 1.0438,1.0602, 1.1305, 1.1468, 1.1533, 1.2260, 1.2707, 1.3423, 1.4149, 1.5709, 1.6017, 1.6083,1.6324, 1.6998, 1.8164, 1.8392, 1.8721, 1.9844, 2.1360, 2.3987, 2.4153, 2.5225, 2.7087,2.7946, 3.3609, 3.3715, 3.7840, 3.9042, 4.1969, 4.3451, 4.4627, 4.6477, 5.3664, 5.4500, 5.7522, 6.4241, 7.0657, 7.4456, 8.2307, 9.6315, 10.1870, 11.1429, 11.2019, 11.4584

The descriptive statistics of the Covid-19 data and the estimates of the parameters of Zech distribution, fitted on the Covid-19 data, as well as the goodness-of-fit criterion and the P-value are presented in Tables 6 and 7 respectively. .

TABLE 6: MLE Estimates, SE, AIC, BIC, AD with the P-Value for the lifetime data

Min	1st Quartile	Median	Mean	3rd Quartile	Max.	Standard Deviation	Skewness	Kurtosis
0.0587	0.4064	1.2484	2.4372	3.3636	11.4584	2.935955	1.702605	5.090844

The results in Table 6 show that the Covid-19 data is skewed. A skewness value of 1.702605 is relatively high, suggesting a significant degree of asymmetry, and the bulk of the data values lie toward the lower end, with a few larger values pulling the mean to the right. The kurtosis value of 5.090844 indicates the distribution has leptokurtic characteristics.

TABLE 7: MLE Estimates, SE, AIC, BIC, AD with the P-Value for the lifetime data

γ	δ	θ	AIC	BIC	AD	P-Value
0.3372 (0.1316)	0.4613 (0.1343)	0.2252 (0.0688)	281.7393	288.7315	0.2418	0.9986

The results in table 7 indicate that Zech distribution fits the Covid-19 data well, having a strong P-value, of 0.9986 which suggests that the distribution of the data is very consistent with the hypothesized distribution.

Figure 2 represents the plots of the empirical and theoretical CDFs, Histogram and theoretical densities, Quantile-Quantile plot and Probability-Probability plot of Zech distribution on the Covid-19 data.

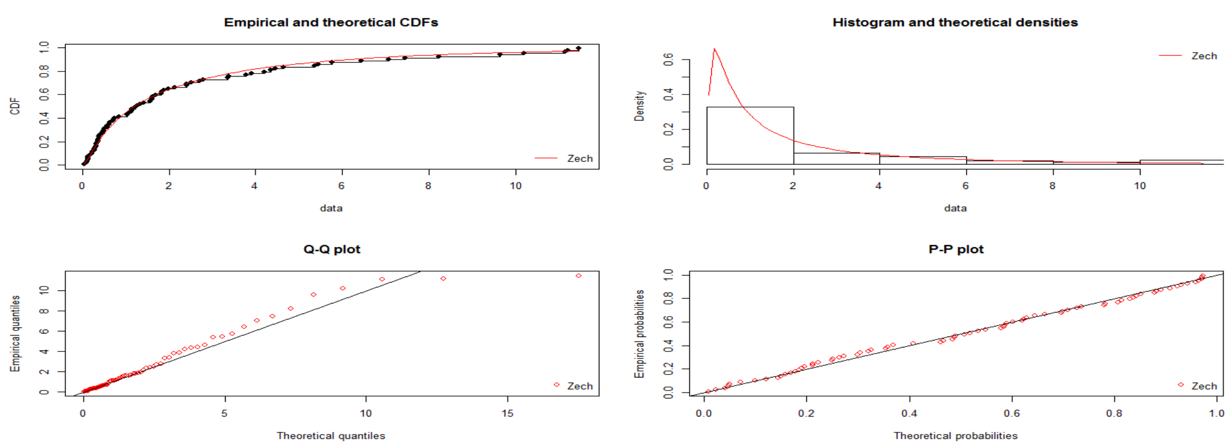


Figure 2: graphs demonstrating the fit of Zech distribution on the Covid-19 data.

3.5 Determination of the True 75th Percentile Failure Rate of the Covid-19 Data Using Zech Distribution

From the estimates of the parameters of Zech distribution, $\hat{\theta} = 0.2252, \hat{\lambda} = 0.3372, \hat{\delta} = 0.4613, q = 0.75$

The qth quantile is given as $t_q = -\frac{1}{\theta}\left\{\ln\left[1-\left(1-\frac{\delta}{\lambda}ln\, q\right)^{-\frac{1}{\delta}}\right]\right\}$

$$t_{0.75} = -\frac{1}{0.2252}\left\{\ln\left[1-\left(1-\frac{0.4613}{0.3372}ln\, 0.75\right)^{-\frac{1}{0.4613}}\right]\right\}$$

$$t_{0.75} = -4.4405 \times \{\ln[1-(1-(1.3680 \times -0.2877))^{-2.1678}]\}$$

$$t_{0.75} = -4.4405 \times \left\{\ln\left[1-\left(1+(0.3935736)\right)^{-2.1678}\right]\right\}$$

$t_{0.75} = -4.4405 \times \{\ln[1-(1.3935736)^{-2.1678}]\}$
$t_{0.75} = -4.4405 \times \{\ln[1-0.4870]\}$
$t_{0.75} = -4.4405 \times ln 0.5130$
$t_{0.75} = -4.4405 \times -0.6675$
$t_{0.75} = 2.9640 \sim 3.0$
The true 75th percentile failure rate of the Covid-19 patients is approximately 3.0

From data 2, the 1st 56 Covid-19 patients out of the total of 76 patients have the failure rate of atmost 3.0. This alone buttresses the fact that Zech distribution is very good at modelling the survival data, since 75th percentile is where $\left(\frac{3}{4} \times N\right) th$ item lies. $\frac{3}{4} \times 76 = 57th$ Item

From the Covid-19 data, the 56th item is 2.7946.
Also, from Table 6, the third quartile is 3.3636. The true 75th percentile failure rate of the Covid-19 patients can be approximated to 3.0.

Table 8: Group ASP for Zech when $\lambda = 0.3372$ and $\delta = 0.4613$ Showing Minimum g and c at 75th Percentile Using the Survival Data

		$r = 5$									$r = 10$								
		$a = 0.5$			$a = 0.7$			$a = 1.0$			$a = 0.5$			$a = 0.7$			$a = 1$		
β	r_2	g	c	P_a	g	c	P_a	g	c	P_a	g	c	P_a	g	c	P_a	g	c	P_a
	2	0	5	0.0000	0	5	0.0000	0	5	0.0000	50	8	0.9606	93	9	0.9611	0	5	
	4	5	3	0.9672	11	4	0.9828	6	4	0.9609	1	4	0.9675	1	5	0.9685	1	0.0000	
0.25	6	2	3	0.9678	1	2	0.9526	2	3	0.9623	1	3	0.9722	1	4	0.9759	1	6	
	8	2	2	0.9892	1	2	0.9808	2	3	0.9867	1	2	0.9531	1	3	0.9660	1	0.9661	
	10	1	1	0.9676	1	2	0.9915	1	2	0.9702	1	2	0.9807	1	3	0.9873	1	5	
																		0.9745	
																		4	
																		0.9675	
																		4	
																		0.9882	
	2	0	5	0.0000	0	5	0.0000	0	5	0.0000	729	9	0.9684	0	5	0.0000	0	5	
	4	40	4	0.9874	18	4	0.9720	-	-	-	3	5	0.9812	4	6	0.9750	4	0.0000	
0.10	6	3	2	0.9522	4	3	0.9800	9	4	0.9886	1	3	0.9722	1	4	0.9759	1	7	
	8	3	2	0.9839	2	2	0.9620	3	3	0.9802	1	2	0.9531	1	3	0.9660	1	0.9734	
	10	3	2	0.9939	2	2	0.9831	3	3	0.9922	1	2	0.9807	1	3	0.9873	1	5	
																		0.9745	
																		4	
																		0.9675	
																		4	
																		0.9882	
	2	0	5	0.0000	0	5	0.0000	0	5	0.0000	949	9	0.9591	0	5	0.0000	0	5	
	4	52	4	0.9837	24	4	0.9628	-	-	-	4	5	0.9751	4	6	0.9750	5	0.0000	
0.05	6	10	3	0.9889	6	3	0.9701	12	4	0.9849	3	4	0.9865	2	4	0.9523	3	7	
	8	4	2	0.9786	6	3	0.9916	3	3	0.9802	1	2	0.9531	1	3	0.9660	1	0.9669	
	10	4	2	0.9919	3	2	0.9747	3	3	0.9922	1	2	0.9807	1	3	0.9873	1	6	
																		0.9856	
																		4	
																		0.9675	
																		4	
																		0.9882	

	2	0	5	0.0000	0	5	0.0000	0	5	0.0000	0	5	0.0000	0	5	0.0000	0	5
	4	80	4	0.9750	-	-	-	-	-	-	6	5	0.9629	7	6	0.9567	7	0.0000
0.01	6	15	3	0.9834	8	3	0.9604	17	4	0.9786	4	4	0.9820	2	4	0.9523	4	7
	8	5	2	0.9733	8	3	0.9888	5	3	0.9672	2	3	0.9856	2	4	0.9882	2	0.9540
	10	5	2	0.9899	4	2	0.9664	5	3	0.9870	2	2	0.9617	2	3	0.9748	2	6
																		0.9808
																		5
																		0.9875
																		4
																		0.9766

Table 8 presents the results of the group acceptance sampling plans based on the truncated life test of Zech distribution's 75th percentile, using the Covid-19 data.

For illustration, assuming a medical practitioner wants to use the proposed ASP in Table 8 to assess whether the 75th percentile failure rate of COVID-19 patients surpasses a specified 75th percentile failure rate. The true 75th percentile failure rate of the Covid-19 patient is 3.0. Assuming a specified 75th percentile failure rate of 0.5, the quantile ratio r_2, is $\frac{3.0}{0.5} = 6$, which implies that the true 75th percentile failure rate of the Covid-19 patients is 6 times its specified 75th percentile rate. Taking the consumer's and producer's risks as 0.01 and 0.05 respectively, i.e. $\beta = 0.01$, and $= 0.05$, $a = 0.7$, $r = 10$ and $r_2 = 6$, the optimal parameters determined from Table 8 are: $g = 2, c = 4$ and $P_a = 0.9523$. To determine the sample size, the fixed $r = 10$ shall be multiplied by $g = 2$, to yield $n = 20$. This means that a sample size of 20 shall be drawn and divided into 2 groups, each consisting of 10 individuals. If no more than four cases fail in each group before the end of the test, it shall be statistically confirmed that the true 75th percentile failure rate is greater than the specified failure rate. If the opposite is true, the lot shall be rejected.

Note:

1. The cells with $g = 0$ indicate that the number of groups is too large, exceeding the set value of 1000, which represents the number of cycles or iterations the plan will run to gather meaningful results. In summary, a value of zero for g means that the plan has not been able to obtain reasonable results after 1000 replications.
2. The cells with (-) indicate that g and c cannot satisfy the conditions.

3.6. Comparison Study

The effectiveness of the proposed Group ASP over those of New Weibull-Pareto and Gompertz distribution is investigated in Table 10 and 11. The plan with the minimum number of groups is adjudged the best.

The estimates of the shape parameters for the Zech distribution, New Weibull-Pareto distribution and Gompertz distribution to be used to develop acceptance sampling plans, using the two datasets are presented in Table 9 below.

Distributions	Estimates of the Shape Parameters on Industrial Data	Estimates of the Shape Parameters on Survival Data
Zech Distribution	$\hat{\gamma} = 0.84740$	$\hat{\gamma} = 0.33720$
	$\hat{\delta} = 0.28560$	$\hat{\delta} = 0.46130$
New Weibull-Pareto Distribution	$\hat{\gamma} = 1.19992$	$\hat{\gamma} = 1.49967$
	$\hat{\delta} = 0.04997$	$\hat{\delta} = 0.79980$
Gompertz Distribution	$\hat{\gamma} = 0.26260$	$\hat{\gamma} = 0.33385$

3.7. Determination of the True Median Lifetime of the Electric Carts Using Zech distribution

$\hat{\theta} = 0.0652, \hat{\lambda} = 0.8474, \hat{\delta} = 0.2856, q = 0.75$, $t_q = -\frac{1}{\theta}\left\{\ln\left[1 - \left(1 - \frac{\delta}{\lambda} ln\, q\right)^{-\frac{1}{\delta}}\right]\right\}$

$$t_{0.5} = -\frac{1}{0.0652}\left\{\ln\left[1 - \left(1 - \frac{0.2856}{0.8474} ln\, 0.5\right)^{-\frac{1}{0.2856}}\right]\right\} = 10.0153 \text{ months} \sim 10 \text{ months.}$$

3.8. Determination of the True Median Failure Rate of the Covid-19 Patients Using Zech Distribution

$\hat{\theta} = 0.2252, \hat{\lambda} = 0.3372, \hat{\delta} = 0.4613, q = 0.25$, $t_q = -\frac{1}{\theta}\left\{\ln\left[1 - \left(1 - \frac{\delta}{\lambda} ln\, q\right)^{-\frac{1}{\delta}}\right]\right\}$

$$t_{0.25} = -\frac{1}{0.2252}\left\{\ln\left[1 - \left(1 - \frac{0.4613}{0.3372} ln\, 0.5\right)^{-\frac{1}{0.4613}}\right]\right\} = 1.1932 \sim 1.2$$

Table 10: Comparison of the Optimal Parameters of the Proposed Group ASP Based on Zech Distribution with the Group ASP Based on NWPD and GOM-D at 10th Percentile for the Industrial Data

		$r = 5$									$r = 10$								
		Zech			GOM-D			NWPD			Zech			GOM-D			NWPD		
				$a = 0.7$										$a = 0.5$					
β	r_2	g	c	P_a	g	c	P_a	g	c	P_a	g	c	P_a	g	c	P_a	g	c	P_a
	2	744	2	0.9627	0	5	0.0000	0	5	0.0000	362	2	0.9731	0	5	0.0000		2	0.9755
	4	45	1	0.9934	499	2	0.9808	60	1	0.9938	32	1	0.9961	142	2	753		1	0.9937
0.25	6	5	0	0.9663	35	1	0.9630	6	0	0.9578	5	0	0.9741	18	1	0.9774	52	0	0.9570
	8	5	0	0.9849	35	1	0.9791	6	0	0.9760	5	0	0.9893	18	1	0.9576	6	0	0.9756
	10	5	0	0.9923	35	1	0.9866	6	0	0.9846	5	0	0.9949	18	1	0.9758	6	0	0.9843
																0.9844	6		
	2	0	5	0.0000	0	5	0.0000	0	5	0.0000	601	2	0.9557	0	5	0.0000	0	5	0.0000
	4	74	1	0.9891	828	2	0.9684	100	1	0.9897	53	1	0.9936	235	2	0.9629		1	0.9896
0.10	6	74	1	0.9986	828	2	0.9906	100	1	0.9979	7	0	0.9639	235	2	86		1	0.9979
	8	8	0	0.9760	57	1	0.9661	9	0	0.9643	7	0	0.9851	29	1	0.9888		0	0.9636
	10	8	0	0.9877	57	1	0.9782	9	0	0.9770	7	0	0.9929	29	1	86		0	0.9765
																0.9613	9		
																0.9750	9		
	2	0	5	0.0000	0	5	0.0000	0	5	0.0000	0	5	0.0000	0	5	0.0000	0	5	0.0000
	4	96	1	0.9859	0	-	0.0000	130	1	0.9866	69	1	0.9917	306	2	0.9520		1	0.9865
0.05	6	96	1	0.9982	0	-	0.0000	130	1	0.9973	9	0	0.9538	306	2	112		1	0.9973
	8	10	1	0.9701	74	1	0.9562	12	0	0.9527	9	0	0.9809	306	2	0.9855		0	0.9517
	10	10	0	0.9847	74	1	0.9718	12	0	0.9694	9	0	0.9909	38	1	112		0	0.9688
																0.9938			
																12			
																0.9674			
																12			
	2	0	5	0.0000	0	5	0.0000	0	5	0.0000	0	5	0.0000	0	5	0.0000	0	5	0.0000
	4	148	1	0.9784	0	-	0.0000	199	1	0.9796	106	1	0.9873	0	-	0.0000		1	0.9794
0.01	6	148	1	0.9972	0	-	0.0000	199	1	0.9959	106	1	0.9987	470	2	172		1	0.9959
	8	16	0	0.9526	0	-	0.0000	199	1	0.9987	14	0	0.9704	470	2	0.9777		1	0.9987
	10	16	0	0.9756	114	1	0.9569	18	0	0.9545	14	0	0.9859	57	1	172		0	0.9536
																0.9905			
																172			
																0.9514			
																18			

Table 10 above shows the comparison of the proposed Group ASP with the Group ASP on the New Weibull-Pareto and Gompertz Distributions for the data on the small electric carts.

3.9. Discussion of Results

Table 10 suggests that Zech distribution, having the lowest number of groups, offers a balanced and efficient alternative to NWPD and GOM-D, particularly for industrial applications**.** The comparative study conducted at the 10th percentile highlights the flexibility of Zech distribution in designing group acceptance sampling plans for any chosen percentile, delivering effective and reliable results.

Table 11: Comparison of the Optimal Parameters of the Proposed Group ASP Based on Zech Distribution with the Group ASP Based on NWPD and GOM-D at 25th Percentile for the Survival Data

		$r = 5$									$r = 10$								
		Zech			GOM-D			NWPD			Zech			GOM-D			NWPD		
		$a = 0.5$									$a = 0.5$								
β	r_2	g	c	P_a	g	c	P_a	g	c	P_a	g	c	P_a	g	c	P_a	g	c	P_a
	2	0	5	0.0000	0	5	0.0000	0	5	0.0000	64	3	0.9712	455	4	0.9564	122	3	0.9669
	4	13	1	0.9891	110	2	0.9801	18	1	0.9723	4	1	0.9853	14	2	0.9726	5	1	0.9668
0.25	6	3	0	0.9549	13	1	0.9616	18	1	0.9916	4	1	0.9983	14	2	0.9916	5	1	0.9897
	8	3	0	0.9818	13	1	0.9782	18	1	0.9964	2	0	0.9758	4	1	0.9712	5	1	0.9956
	10	3	0	0.9918	13	1	0.9860	3	0	0.9529	2	0	0.9891	4	1	0.9813	5	1	0.9977
	2	0	5	0.0000	0	5	0.0000	0	5	0.0000	106	3	0.9528	0	5	0.0000	0	5	0.0000
	4	21	1	0.9824	182	2	0.9673	29	1	0.9558	6	1	0.9781	23	2	0.9554		2	0.9921
0.10	6	21	1	0.9980	182	2	0.9903	29	1	0.9865	6	1	0.9975	23	2	35		1	0.9836
	8	4	0	0.9758	21	1	0.9651	29	1	0.9942	2	0	0.9758	6	1	0.9863	8	1	0.9929
	10	4	0	0.9891	21	1	0.9775	29	1	0.9970	2	0	0.9891	6	1	0.9571	8	1	0.9963
																0.9721	8		
	2	0	5	0.0000	0	5	0.0000	0	5	0.0000	917	4	0.9794	0	5	0.0000	0	5	0.0000
	4	27	1	0.9774	236	2	0.9578	384	2	0.9924	8	1	0.9709	145	3	0.9865		2	0.9898
0.05	6	27	1	0.9975	236	2	0.9875	38	1	0.9823	8	1	0.9967	30	2	45		1	0.9775
	8	5	0	0.9699	28	1	0.9537	38	1	0.9924	3	0	0.9640	30	2	0.9821		1	0.9903
	10	5	0	0.9864	28	1	0.9701	38	1	0.9961	3	0	0.9837	8	1	11		1	0.9950
																0.9923			
																11			
																0.9630			
																11			
	2	0	5	0.0000	0	5	0.0000	0	5	0.0000	0	5	0.0000	0	5	0.0000	0	5	0.0000

	4	41	1	0.9659	0	-	0.0000	589	2	0.9884	12	1	0.9566	222	3	0.9794	2	0.9845
0.01	6	41	1	0.9962	363	2	0.9808	58	1	0.9731	12	1	0.9950	45	2	69	1	0.9674
	8	8	0	0.9523	363	2	0.9919	58	1	0.9885	4	0	0.9522	45	2	0.9733	1	0.9859
	10	8	0	0.9784	42	1	0.9555	58	1	0.9941	4	0	0.9784	45	2	16	1	0.9927
																0.9885		
																16		
																0.9941		
																16		

Table 11 compares the Group Acceptance Sampling Plans (GASPs) for three distributions (Zech, Gompertz (GOM-D), and New Weibull-Pareto (NWPD)) under various conditions, particularly at the 25th percentile for the Covid-19 data.

3.10. Discussion of Results

The Table shows that at the 25th percentile, the Zech distribution stands out as a more flexible and efficient option for group acceptance sampling plans compared to the New Weibull-Pareto and Gompertz distributions. It achieves an optimal balance between reliability and cost-effectiveness by providing the smallest number of groups across the evaluated parameters.

The failure probability derived from the Zech distribution is employed to design group acceptance sampling plans (ASP) based on its 10th, 25th, 50th, and 75th percentiles. These plans are applied to simulated data, arbitrary parameter values, as well as industrial and survival datasets.

Tables 1 and 2 present the results of the group ASP developed for the truncated life test using the percentiles of the Zech distribution with simulated data and arbitrary parameter values, respectively. The findings reveal an inverse relationship between consumer risk and the number of groups.

Tables 3 and 4 provide descriptive statistics and parameter estimates of the Zech distribution for the industrial data. The data shows some degree of asymmetry, and a p-value of 0.9772 indicates a strong fit of the Zech distribution. The application of the proposed plan to the industrial data is summarized in Table 5.

For the survival data, Tables 6 and 7 detail the descriptive statistics and parameter estimates of the Zech distribution. The Covid-19 survival data is skewed, and a p-value of 0.9986 confirms an excellent fit of the Zech distribution. The results of applying the proposed plan to the survival data are shown in Table 8.

To facilitate comparison, Table 9 presents the shape parameter estimates for the Gompertz and New Weibull-Pareto distributions. Tables 10 and 11 highlight the results of a comparative study on industrial and survival data, respectively. In each case, the proposed group ASP with the smallest number of groups is identified as the most effective and cost-efficient.

Figures 1 and 2 illustrate the empirical and theoretical cumulative distribution functions (CDF), histograms and theoretical densities, P-P plots, and Q-Q plots for the Zech distribution fitted to the industrial and survival datasets. Figure 3 displays the operating characteristic (OC) curves for the three distributions, showing that the ASP based on the Zech distribution, with the steepest slope, is the most accurate of the plans.

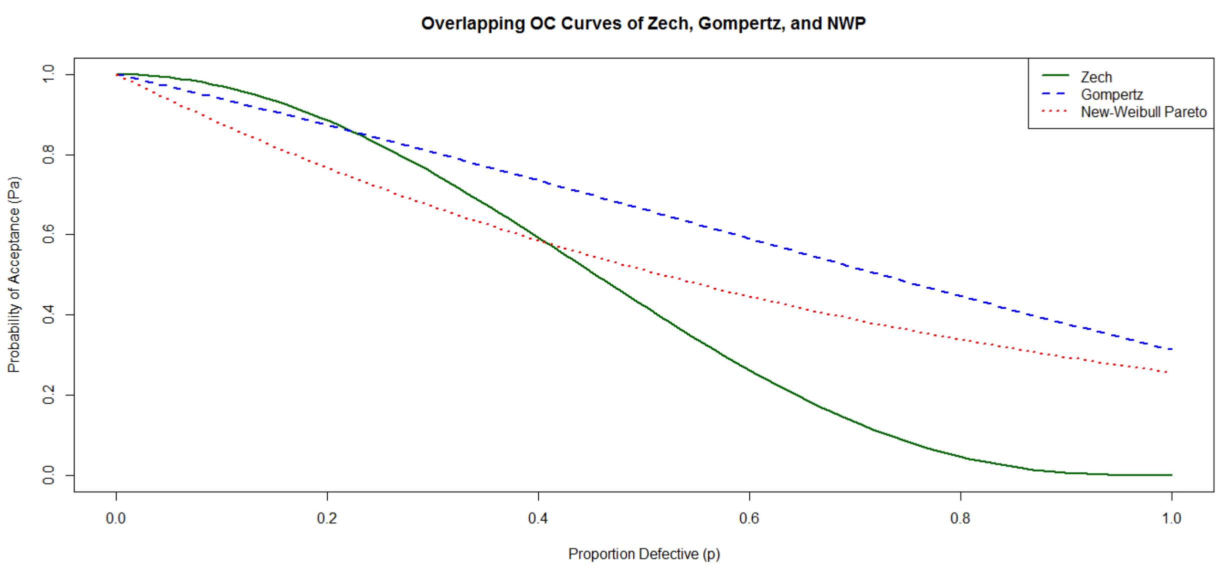


Figure 3: The operating characteristics curves of Zech, Gompertz, and New Weibull-Pareto distributions.

The Steeper, the better. The ASP based on Zech distribution has the steepest slope, making it the best of the three sampling plans.

4.0 Conclusion

A group acceptance sampling plan is developed for the truncated life tests based on the percentiles of Zech distribution. The table patterns of the proposed plans were investigated and applied to both simulated data and arbitrary parameter values. Also, to show the flexibility of Zech distribution for developing acceptance sampling plans, two datasets comprising industrial data and survival data were used to design the group ASP. Comparative studies are carried out on the proposed group ASP and the group acceptance sampling plans developed for the Gompertz and New Weibull-Pareto distributions. The results showed that the group ASP on Zech distribution is the more efficient and cost-effective by providing the smallest number of groups.

Funding: There is no funding for this research.

Conflict of Interest: The authors declare that they have no conflict of interest.

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