A t-Distribution-Based Particle Filter for Bearings-Only Tracking

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Abstract
Bearings-only target tracking is a nonlinear estimation problem often addressed by linearised filters where the uncertainty in the sensor and motion models is typically modeled by Gaussian densities. In this paper, a particle filter or sequential Monte Carlo method is developed, based on student-t distribution, which is heavier tailed than Gaussian's and hence more robust. The t-distribution-based particle filter provides an approximate solution to nonlinear non-Gaussian estimation problems. To estimate the target state based on samples, an expectation maximisation (EM)-type algorithm was developed and embedded in a student-t particle filter. The expectation step was implemented by the particle filter. In this step, the distribution of the states and the state vector were estimated. Consequently, in the maximisation step, the nonlinear observation equation was approximated as a mixture of the Gaussian and student-t models. A bearings-only tracking problem was simulated to present the implementation of the particle filter algorithm based on both the mixture of the Gaussian model and student-t. Simulations and real life data taken from the digital global system for mobile communications (GSM) real-time data-logging tracking system showed that the student-t-based particle filter significantly outperformed the Gaussian mixture filter and successfully accommodated a nonlinear model for a target-tracking scenario.

Keywords: bearing-only tracking, expectation maximisation, particle filter, student-t distribution

Introduction

Bearings-only target tracking is the problem of generating an inference engine on the state of a target using a sequence of observations in time, which is to recursively estimate the probability density function of the target state. It is a nonlinear estimation problem often addressed by linearised filters modelled by Guassian densities. This problem has attracted interest over past decades as it arises in many modern applications, including: robots localisation, visual tracking, radar tracking and satellite navigation.

In addition, it involves tracking an object (typical examples include ships, planes and other moving vehicles). A successful target-tracking depends on an effective extraction of the useful information about the target state from available observations. The tracking problem can be solved by recursively calculating some degrees of belief in the target state, taking different values, given available observations. Thus, a construction of the conditional probability density function of the target state is generally required. Since the target state uncertainty and the measurement-originated uncertainty are the two major unavoidable obstacles for target tracking, a good model of the target motion will effectively facilitate the design of the required tracking algorithm.

This paper focuses on a student-t distribution-based particle filter, which is heavier tailed than Gaussians and provides an approximate solution to nonlinear non-Gaussian estimation problems.

The commonly used models are the state-space model described in the following forms:

$$x_t = f(x_{t-1}) + w_t \tag{1}$$

$$y_t = g(x_t) + v_t \tag{2}$$

where f and g are either linear or nonlinear functions, w_t and v_t represent the independent and identically distributed (i.i.d) process and measurement noise sequence, respectively. x_t and y_t denote the target state vector and the measurement at sample time t, respectively. Models represented by equations (1) and (2) are referred to as state-space models. This includes such models as the bearings-only tracking model.

Target-tracking using bearings-only measurements is a difficult task due to the unobservability of elements of the target's state and high degree of the nonlinear measurement process (La Scala *et al.,* 2007). The filtering algorithms involve a nonlinear measurement process, which when linearised can lead to time-varying parameter biases as explained by Aidala (1979). The common estimation algorithms used for bearings-only target-tracking are: Least Squares (batch and recursive forms), Maximum Likelihood

Estimator, Extended Kalman Filter (EKF) and Particle Filters or Bayesian Methods.

Most researchers in the field of bearings-only tracking have concentrated on tracking a non-manoeuvring target (Aidala and Hammel, 1983; Robinson and Yin, 1994; Bar-Sharlom *et al.*, 2001; Clark *et al.*, 2007). Due to inherent nonlinearity and observability issues, it is difficult to construct a finite-dimensional optimal filter even for this relatively simple problem. As for the bearings-only tracking of a manoeuvring target, the problem is much more difficult.

Early research projects focused mainly on analytical derivations for the observability criteria of the estimation process, comparisons of the convergence properties and performance of the different types of method used for target-tracking. Since bearings-only target estimation involves a nonlinear measurement process, filtering and observability complications arise. Lindgren and Gong (1978) analysed the observability associated with a least-squares estimation approach and showed that for a constant velocity target and a constant velocity vehicle moving in a 2-D plane, the target estimation is unobservable until the vehicle executes a manoeuvre. Kalman Filtering as a method of estimation was used by Nardone *et al.* (1984).

An extended Kalman filter (EKF) approach needs to be used instead of the Kalman filter since the bearings-only estimation problem involves nonlinear measurements. The EKF is one of the most widely used methods but it is unable to relinearise the nonlinear system when new measurements become available and therefore gives an unsatisfactory performance. This has given rise to refinements of the EKF; for example, modified polar coordinates EKF (Aidala and Hammel, 1983) and the shifted Rayleigh filter (SRF) (Clark *et al.*, 2007). However, both of these EKF variants can only track a single mode of the posterior probability density function of the target state.

A multi-hypothesis EKF (MHEKF) was proposed by Kronhamn (1998) to track the multiple hypothesis of target state. The MHEKF described by Kronhamn (1998) determines a fixed number of EKFs at the first available measurement. This idea was extended by Musicki (2009) so that the filter bank can dynamically change its size at each time step based on the current measurement likelihood. Nerurkar *et al.* (2009) and Huang *et al.* (2010) proposed bank of maximum, a posteriori (MAP) estimator, which selects most probable hypotheses of the target trajectory based on optimality at each time step.

A pseudolinear filter formulation was proposed by Aidala and Nardone (1982). It attempts to linearise the dynamics and measurement models. However by linearising the dynamics, the noise becomes non-Gaussian, which when propagated through the filter causes estimation bias.

For the bearings-only tracking problem, the bias is introduced only in the position estimate and is highly dependent on the geometry of the vehicle manoeuvres, suggesting that the estimation performance can be improved by the proper design of the vehicle trajectory. Musicki and Evans (2006) and Musicki (2007) examined the effects of nonlinearity and observability on the degree of difficulty of the single-sensor bearings-only tracking problem by a Gaussian sum measurement approximation filter. Aidala and Hammel (1983) proposed the modified polar coordinates (MPC) filter. The filter uses an EKF algorithm with a state vector choice based on polar coordinates that attempts to separate the observable and unobservable components of the estimated state by using a different coordinate system. The resulting filter is stable and asymptotically unbiased. The modified polar coordinate filter shows the dependence of the target estimation on the vehicle manoeuvres; once again suggesting that the estimation can be improved by designing a good trajectory.

Lately, a variant of the MPC system, called the log polar coordinate (LPC) basis, was proposed by Brehard and Le Cadre (2006). As pointed out by Brehard and Le Cadre (2006), an advantage of LPC over MPC is that an EKF using LPC is more robust than the one that makes use of MPC (La Scala *et al.*, 2007). De Vlieger (1992) used a piecewise linear model of the target motion and a Maximum Likelihood Estimator (MLE) approach for target tracking. He used numerical methods to condition the measurement model to increase the observability of the estimation. Goshen-Meskin and Bar-Itzhack (1992) derived the observability requirements for piecewise constant linear systems. Tao *et al.* (1996) showed that for a MLE approach, it is important to consider the correlation of the noise and that ignoring it degrades the performance of the estimation.

Several modifications to the classical estimation algorithms have been explored. Some researchers have attempted to smoothen the trajectory within the constraints of a known target behaviour model. Other efforts consist of designing multiple filters for different known target scenarios and using the statistical properties of the innovation to switch between the algorithms. Another approach has been to support multiple Kalman filters simultaneously and develop an estimate by combining all the filters. Later work by Bar-Shalom *et al.* (2002) has focused on using interacting multiple models (IMM). These algorithms employ a constant velocity (CV) model along with manoeuvre models to capture the dynamic behaviour of a manoeuvring target scenario. Le Cadre and Tremois (1998) modelled the manoeuvring target using the CV model with Gaussian noise and developed a tracking filter in the hidden Markov model framework. Radhakrishnan *et al.* (2010) proposed the coordinated turn (CT) model along with the EKF to track all possible dynamics such as velocity, acceleration and coordinated turns of the manoeuvring targets.

Particle filtering or sequential Monte Carlo techniques have also been explored by Liu *et al.* (2002), Ristic *et al.* (2004), Clark *et al.* (2007), Gustafsson (2010), Haluk (2011) and Dosso and Wilmut (2013). Particle filters have the advantage of being able to deal with nonlinear systems and non-Gaussian noise models making them particularly well-suited to bearings-only tracking. Researchers in the field of bearings-only tracking have shown that particle filters can also accommodate unknown and stochastic target models, making them more versatile than classical filters (Liu and Hao, 2013; Li and Zhou, 2013; Tirri *et al.*, 2014; Warner *et al.*, 2015; Li *et al.*, 2015). However, they require increased computational resources and, for fast convergence, need a fairly accurate description of the measurement likelihood function and a good initial distribution on the estimated target location.

Bearings-only tracking involves estimating the target states based on angle measurements at a sensor node. The target is assumed to move in the *x–y* plane and to follow a constant-velocity motion model (Bar-Shalom and Fortmann, 1988) with a state update period of 1 s. The state vector X_t contains the positions and velocities of the object in the *x–y* directions, respectively: the target state at time t is $X_t = (x_t, y_t, \dot{x}_t, \dot{y}_t)^T$, where x_t and y_t represent the target positions and $\dot{x}_t$ and $\dot{y}_t$ represent the corresponding velocities. One possible discretisation of this model is given by Gordon *et al.* (1993):

$$X_t = \emptyset x_{t-1} + \Gamma\, u_t \qquad t = 1, \cdots, N \tag{3}$$

The equation of the observed bearing, z_t, is

$$z_t = tan^{-1}\left(\frac{y_t}{x_t}\right) + w_t \tag{4}$$

where $u_t = (u_x, u_y)^T$, $\emptyset = \begin{pmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{pmatrix}$ and

$$\Gamma = \begin{pmatrix} \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & \frac{T^2}{2} \\ 0 & T \end{pmatrix}$$

u_t represents system noise and is Gaussian distributed with covariance $\sum_u \sigma_u^2 I_2$ where I_2 is a 2 x 2 identity matrix, T = 1 s is the normalised sampling period, z_t is the observed bearing of the object measured by the sensor at time t and w_t represents a Gaussian measurement noise with mean zero and variance σ_w^2.

Before measurements are taken, the particle filter recursion is started with initial state vector in the form of a four-dimensional Gaussian variable with known mean and covariance matrix. The model is four-dimensional and nonlinear due to a transcendental function in the observation equation. It has been shown through intense simulations, by Gordon *et al.* (1993), that particle filters are much more efficient for this problem than the traditional EKF. The bearings-only tracking (BOT) problem is illustrated in Figure 1.

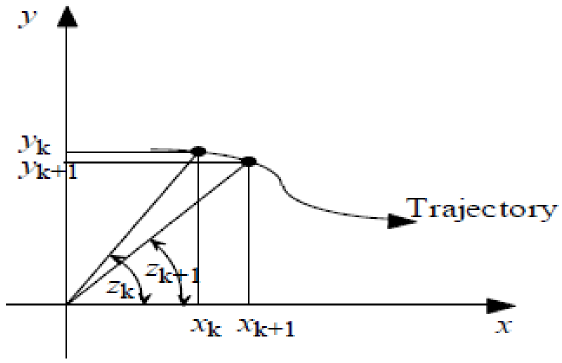

Figure 1: The Bearings-Only Tracking (BOT) problem

In this paper, the particle filter is developed based on the student t-distribution for nonlinear bearing-only tracking problem and compared with the normal mixture-based particle filter of Kim and Stoffer (2008).

Methods of Estimation

The Expectation-Maximisation Algorithm

State estimation in a nonlinear state-space dynamical system consists of estimating the state data vector using a sequence of noisy measurements given by the model in equation (2). The main idea in EM-based algorithms is to solve the state estimation problem in the presence of model uncertainty in two iterative steps. Starting from some initial parameters $\theta^{(0)}$ the algorithm iteratively applies:

E-step: Compute the expected likelihood, $Q(\theta|\theta^{(k)})$

$$Q(\theta|\theta) = E(\log f\,(x|\theta)|y, \theta)$$

M-step: Choose $\theta^{(k+1)}$, the parameter values that maximise the function, $Q(\theta|\theta^{(k)})$.

In the E-step of the proposed algorithm, an approximation of the desired distribution of the states given in the measurements is formulated. This distribution is then used to estimate the states. In nonlinear systems this conditional density is generally non-Gaussian and can be quite complex. An SMC (particle filter) algorithm (Doucet *et al.*, 2001) was used to estimate and recursively update this distribution in time. This aided the convergence of the algorithm to the global optimum. In the maximisation (M) step, the unknown measurement process was approximated by fitting the observations to a student-t model using the current estimates of the states.

Sequential Monte Carlo Methods

Since the introduction of SMC in the 1960s, it has become an emerging methodology for the nonlinear or non-Gaussian state-space models. SMC methods or particle filters are a class of recursive simulation methods for solving filtering problems (Doucet *et al.*, 2001; Gordon *et al.*, 1993).

Let $\{x_{0:t-1}^{(i)},\ w_{0:t-1}^{(i)}\ i = 1, \cdots, N\}$ be samples and associated weights approximating the density function $p(x_{0:k-1}|y_{0:k-1}), \{x_{0:t-1}^{(i)}\}_{i=1}^{N}$ is a set of particles with associated weights $\{w_{0:t-1}^{(i)}\}_{i=1}^{N}$ with $\sum_{i=1:N} w_{t-1}^{(i)} = 1$, then the density function is approximated by

$$p(x_{0:k-1}|y_{0:k-1}) \approx \sum_{i=1}^{N} w_{t-1}^{(i)}\, \delta(x_{t-1} - x_{t-1}^{(i)})$$

$\delta(x)$ signifies the Dirac delta role.

The particle approximation $\{w_t^{(i)},\ x_t^{(i)}\}_{i=1}^{N}$ is also transformed into an equally weighted random sample from $p(x_{0:k-1}|y_{0:k-1})$ by sampling, with replacement from the discrete distribution $\{w_t^{(i)},\ x_t^{(i)}\}_{i=1}^{N}$.

Particle Filter Algorithm

Kitagawa and Sato (2001) and Kitagawa (1996) gave an algorithm for filtering in general state-space models thus:

Monte Carlo filtering for general state-space models

1. For $i = 1, \cdots, N$, generate a random number $f_0^{(i)} \sim p\,(x_0)$
2. Repeat the following steps for $t = 1, \cdots, T$:
   a. For $i = 1, \cdots, N$, generate a random number $w_t^{(i)} \sim q\,(w)$
   b. For $i = 1, \cdots, N$, compute $p_t^{(i)} = F(\,f_{t-1}^{(i)}, w_t^{(i)})$
   c. For $i = 1, \cdots, N$, compute $w_t^{(i)} = p(\,y_t|p_t^{(i)})$
   d. Generate $f_t^{(i)},\ i = 1, \cdots, N$ by resampling $p_t^{(i)}, \cdots,\ p_t^{(N)}$
3. This Monte Carlo filter returns $\{f_t^{(i)},\ i = 1, \cdots, N,\ t = 1, \cdots, m\}$ so that $\sum_{i=1}^{N} \frac{1}{N}\delta(x_t - f_t^{(i)}) \approx f(x_t|Y_t)$.

Particle Smoothing Algorithm

Let $\{s_t^{(i)}, w_t^{(i)}\}_{i=1}^{N}$ be a set of particle smoothers and associated weights approximating the density function $f(x_t|Y_n)$, then the density function is approximated by $f(x_t|Y_n) \approx \sum_{i=1}^{N} w_t^{(i)}\, \delta(x_t - s_t^{(i)})$.

The problem with smoothened estimates is degeneracy. Godsill *et al.* (2004) suggested a new smoothing method (called particle smoother using backwards simulation). The method assumes that the filtering has already been performed. The particles and associated weights, $\{f_t^{(i)}\}_{i=1}^{M}$, $\{w_t^{(i)}\}_{i=1}^{N}$ can thus approximate the filtering density by

$$f(x_t|Y_t) = \frac{\sum_{i=1}^{N} w_t^{(i)} \delta(x_t - f_t^{(i)})}{\sum_{i=1}^{N} w_t^{(i)}}$$

The following is the algorithm from Godsill *et al.* (2004):

Particle smoother using backwards simulation

Suppose weighted particles $f_t^{(i)}, w_t^{(i)}, i = 1, \cdots, N$ are available for $t = 1, \cdots, n$. For $i = 1, \cdots, M$,

1. Choose $s_n^{(i)} = f_n^{(j)}$ with probability $w_n^{(j)}$
2. For $n - 1$ to 1
   a. Calculate $w_{t|t+1}^{(j)} \propto w_t^{(j)} f(s_{t+1}^{(i)} \,\big|\, f_t^{(j)})$ for each j
   b. Choose $s_t^{(i)} = f_t^{(j)}$ with probability $w_{t|t+1}^{(j)}$
3. $s_{1:n}^{(i)} = (s_1^{(i)},\ s_n^{(i)})$ is an approximate realisation from $p(X_n|Y_n)$.

Sequential Monte Carlo Expectation Maximisation (SMCEM) for Bearings-Only Tracking

The entire procedure based on student-t distribution consists of three main steps: filtering, smoothing and estimation. With the output of filtering and smoothing step, an approximate expected likelihood is then calculated.

Filtering Step

The algorithm below for the filtering and smoothing steps shows an extension of the results of Godsill *et al.* (2004) and Kim and Stoffer (2008).

1. Generate $f_0^{(i)} \sim N(\mu_0,\ \sigma_0^2)$
2. For $t = 1, \cdots, n$
   a. Generate a random number $w_t^{(i)} \sim N(0, \tau), i = 1, \cdots, M$
   b. Compute $p_t^{(i)} = \emptyset f_{t-1}^{(i)} + w_t^{(i)}$
   c. Compute $$w_t^{(i)} = p(y_t|p_t^{(i)}) \propto e^{-\frac{x_t}{2}}\left(1 + \frac{y_t^2\, e^{-x_t}}{v-2}\right)^{-\frac{v+1}{2}}$$
   d. Generate $f_t^{(i)}$ by with weights, $w_t^{(i)}$.

Smoothing Step

Suppose that equally weighted particles $\{f_t^{(i)}\}, i = 1, \cdots, M$ from $f(x_t|Y_t)$ are available for $t = 1, \cdots, n$ from the filtering step:

1. Choose $[s_n^{(i)}] = [f_n^{(j)}]$ with probability $\frac{1}{M}$

2. For $n-1$ to 0
   a. Calculate

$$w_{t|t+1}^{(j)} \propto f\left(s_{t+1}^{(i)} \middle| f_t^{(j)}\right) \propto exp\left(-\frac{(s_{t+1}^{(i)} - \emptyset\, f_{t-1}^{(i)})^2}{2\tau}\right) \frac{1}{\sqrt{\pi\,(v-2)}} \frac{\Gamma\left[\frac{v+1}{2}\right]}{\Gamma\left(\frac{v}{2}\right)} e^{-\frac{s_{t-1}}{2}} \left(1 + \frac{y_t^2\, e^{-s_{t-1}^{(j)}}}{v-2}\right)^{-\frac{v+1}{2}} \quad \text{for each } j$$

   b. Choose $\left[s_t^{(i)}\right] = \left[f_t^{(j)}\right]$ with probability $w_{t|t+1}^{(j)}$
3. $s_{0:n}^{(i)} = \{(s_0^{(i)}, \cdots, s_n^{(i)})\}$ is the random sample from $f(x_0, \cdots, x_n | Y_n)$
4. Repeat steps (1)–(3) for $i = 1, \cdots, M$ and calculate

$$\acute{x}_t^n = \frac{\sum_{i=1}^M s_t^{(i)}}{M}, \quad \acute{p}_t^n = \frac{\sum_{i=1}^M (s_t^{(i)} - \acute{x}_t^n)^2}{M-1},$$

$$\acute{p}_{t,t-1}^n = \frac{\sum_{i=1}^M (s_t^{(i)} - \acute{x}_t^n)(s_{t-1}^{(i)} - \acute{x}_{t-1}^n)}{M}$$

$$E\left[1 + \frac{y_t^2\, e^{x_t}}{v-2}\right]^{-\frac{v+1}{2}} = \frac{n\,(v-2)}{(v+1)\sum_{i=1}^n y_t^2\; e^{-y_t + v_t} \left[1 + \frac{y_t^2\, e^{x_t}}{v-2}\right]^{-1}}$$

Estimation Step

Herein, $(x_0, \cdots, x_n)$ is viewed as unobserved and the EM algorithm is applied. The procedure performed in this algorithm consisted of running a filtering and smoothing step for the given parameters.

Using equations (3) and (4), the proposed technique for target-tracking was applied. The state update was used to propose new particles. This provided a sub-optimal recursive estimate of the target position in the $x - y$ plane. As the target is observed in its motion, new data z_t accumulate, along with new parameters $\ddot{x}_t, \ddot{y}_t$. The vector of the unknown at time t is $\theta_t = x_1, y_1, \ddot{x}_1, \ddot{y}_1, \cdots, \ddot{x}_t, \ddot{y}_t$, and the data are $z_{1:t} = (z_1, \cdots, z_t)$. Therefore, the target distribution evolves in an expanding space, $\boldsymbol{\Omega_t}$.

As t increases, the aim here is to maintain a set of sampled particles in $\boldsymbol{\Omega_t}$, which can be used to estimate aspects of the distribution of interest. In particular, these particles are used at any given time point t to approximate the conditional distribution for the current state of the object, given that the data, $z_{1:t}$ accumulated up to that point. The procedure for the BOT problem is summarised below:

Given the observed data z_t at t

For $i = 1, 2, \cdots, N$ sample particles, $z_t^{(i)}$ are drawn from the density

$X_t^{(i)} \sim p\,(X_t^{(i)} | X_{t-1}^{(i)})$ using equation (3):

$$x_t^{(i)} = x_{t-1}^{(i)} + \dot{x}_{t-1}^{(i)} + u_{x_t}^{(i)}, \quad \dot{x}_t^{(i)} = \dot{x}_{t-1}^{(i)} + u_{x_t}^{(i)}, \quad y_t^{(i)} = y_{t-1}^{(i)} + \dot{y}_{t-1}^{(i)} + u_{y_t}^{(i)}, \quad \dot{y}_t^{(i)} = \dot{y}_{t-1}^{(i)} + u_{y_t}^{(i)}$$

The weights are updated recursively using

$$\widehat{w}_t^{(i)} = w_{t-1}^{(i)}\, p\,(z_t | x_t^{(i)})$$

where $z_t | x_t^{(i)} \sim t\ distribution\ (v)$.

Subsequently, evaluating this distribution at time t for the parameters estimation, by using the EM algorithm and SMC, and then calculating the output:

$$\dot{x}_t = \sum_{i=1}^N w_t^{(i)}\, x_t^{(i)}, \qquad \dot{x}_t^{(i)} = \sum_{i=1}^N w_t^{(i)}\, \dot{x}_t^{(i)},$$

$$\dot{y}_t = \sum_{i=1}^N w_t^{(i)}\, y_t^{(i)}, \qquad \dot{y}_t^{(i)} = \sum_{i=1}^N w_t^{(i)}\, \dot{y}_t^{(i)}$$

Therefore, the mean estimate of the target state and the covariance matrix of the estimate error are approximated by: $\mu_t = \sum_{i=1}^N w_t^{(i)}\, x_t^{(i)}$,

$$\Sigma_u = \sum_{i=1}^N w_t^{(i)} (x_t^{(i)} - \mu_t)(x_t^{(i)} - \mu_t)^T$$

Results

A bearings-only tracking problem was simulated to present the implementation of the SMCEM algorithm based on both the mixture of Gaussian's model of Kim and Stoffer (2008) and the student-t. A target trajectory and associated measurements was generated according to equations (5) and (6) with the parameter values $\sqrt{\sigma_u^2} = 0.001$, $\sqrt{\sigma_w^2} = 0.005$, the initial state of the target $X_0 = (5, 5, 1, 0)^T$ and covariance $= diag(7.5, 2.55, 5.5, 2.7)$T. The time between the successive measurements was $T = 1\,s$ and a single bearing measurement was obtained in each time step.

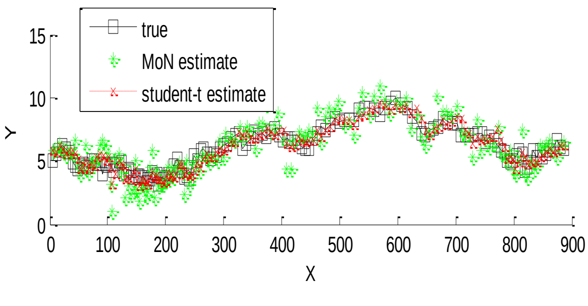

Figure 2: Three scenarios for the BOT: Representation of the trajectories of the true target path (shown by squares), MoN (shown by asterisks) and the student-t estimate (shown by dotted lines)

Figure 2 gives the true target path in the $x - y$ plane, with the position of the target at each time being shown by a square and the mixture of normal by an asterisk. The result of applying the SMCEM with $N = 2000$ particles is shown in Figure 2. The number of particles was chosen such that further increase in N does not bring any significant improvement in the tracking performance. The cross symbol gives the student-t estimates such that the estimate moves towards the true target path. The performance is evaluated using the mean square error (MSE) for each time, according to Sanjeev *et al.* (2004).

$$MSE(t) = \frac{1}{N}\sum_{t=1}^{N}(x_t^{true} - x_t)^2 \quad (5)$$

where x_t denotes the estimate at time t, N is the total number of realisations over which the MSE is averaged. The MSE values (11.0112 and 7.2197 for mixture normal and student-t, respectively) are obtained independently for each element of the state in the BOT problem. For the student-t-based filter, no tracks diverged. It can be seen that the accuracy of the position estimation of the student-t particle filter is significantly higher than that of the normal mixture.

Dynamic Modeling of a Vehicle Tracking

The proposed estimation technique was also applied to the problem of tracking a moving vehicle. Data was taken from the digital GSM real-time data logging tracking system. From the data collected, the vehicle's position $X_t = (x_t, y_t)$ was estimated at time t, and its velocity v_t. Also at each time step a new measurement z_t was obtained.

The velocity evolved over time according to

$$p(v_t|v_{t-1}) \quad (6)$$

The vehicle moved, based on the evolved velocity, according to a dynamics model:

$$p(X_t|X_{t-1},\ v_t) \quad (7)$$

The measurements were governed by a measurement model: $p(Z_t|X_t)$ (8)

And the measurement likelihood factor was

$$p(Z_t|X_t) = \prod_{i=1}^{N}(Z_t|X_t) \quad (9)$$

At each time step t, an estimate of the proposed technique about the tracked vehicle trajectory and velocity was produced based on measurements:

$$M_t = p(X_t\,, v_t \mid Z_t) \quad (10)$$

Equation (10) encodes the vehicle motion and was approximated using the student-t distribution. The measurement update was carried out by computing the importance weights w_t for all the particles.

The results for this case are presented in Figure 3. The student-t-based SMCEM algorithm was able to track the true path of the vehicle being tracked and remained stable and converged. This therefore shows that the accuracy and convergence of the estimation is improved by increasing the information provided by the measurements.

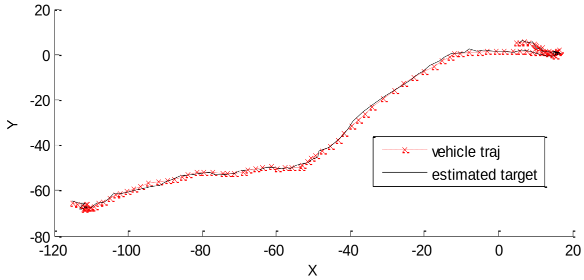

Figure 3: Estimates of the vehicle being tracked

Discussion

In this paper, a student-t particle filter algorithm for solving problems for nonlinear non-Gaussian state-space estimations when the observation model is uncertain is proposed. The goal was to considerably improve the tracking performance of a constrained tracking scenario using student-t particle filter, which is heavier tailed than Gaussian's and hence more robust. A bearings-only tracking (BOT) problem was simulated to present an implementation of the SMCEM algorithm based on both the mixture of the Gaussian's model of Kim and Stoffer (2008) and the student-t.

The simulated results, shown in Figure 2, for three scenarios for the BOT – the representation of the trajectories of the true target path (shown by a square), MoN (shown by an asterisk) and the student-t estimate (shown by dotted lines) – infers that the estimation of the student-t particle filter significantly outperformed that of the Gaussian's mixture filter.

The proposed estimation technique was also applied to the problem of tracking a moving vehicle. The results presented in Figure 3 reveals that the student-t-based SMCEM algorithm is able to track the true path of the vehicle being tracked, remains stable and also converges. Thus, the proposed estimation technique is capable of approximating a wide range of non-linearities in the measurement and state transition processes. Also, implementing the E-step with a particle filter provides the possibility of employing the algorithm in the presence of non-Gaussian noise.

Conclusion

It has been shown, in this paper, that a student-t distribution-based particle filter provides a much better performance than the normal mixture-based particle filter. An EM-type algorithm for solving a joint estimation–identification problem for nonlinear non-Gaussian state-space estimation when the observation model is uncertain is proposed. The expectation (E) step is implemented by the particle filter. Within this step, the distribution of states, given the measurements as well as the state vectors, is estimated. Consequently, in the maximisation (M) step, the nonlinear measurement process parameters are approximated as a mixture of normal model and as a student-t model. The SMCEM algorithm based on both the mixture of Gaussian's model of Kim and Stoffer (2008) and the student-t was used to solve a nonlinear bearing-only tracking problem. It has been shown that the accuracy of the position estimation of the student-t filter is significantly higher than that of the normal mixture. Additionally, the method was applied to real life data taken from the digital GSM real-time data logging tracking system. It was again shown that the student-t-based algorithm was successful in accommodating a nonlinear model for a target-tracking scenario.

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